Skew index: descriptive analysis, explanatory power and short-term forecast.

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Abstract

This paper analyzes the behavior of the *Skew Index,* presenting a series of stylized facts (empirical characteristics) given the analysis of different stock market indexes vastly employed in the financial literature as reference of the financial market (S&P500 and MSCI World indexes). Moreover, Skew Index is compared with investor *fear indexes*, such as the CBOE VIX, IVX, XAU (Gold and Silver) and the Bull-Bear Spread. Data length employed from the *Chicago Board Options Exchange® Skew Index* consists of monthly information from January, 1990 to December, 2018. The Skew Index is calculated from the S&P500's tail risk price, specifically from the OTM options prices of this stock index, which reveals a large compensation variable in time due to fear of financial disasters. Finally, a model is developed that allows forecasting events of the Skew Index in the short term, with ARIMA and GARCH processes.

1. Introduction

Several approaches have been employed to analyze financial crashes, among which we find conditional volatility models as the most usual in the literature. This paper aims to study the CBOE Skew Index ("Skew Index"), testing the hypothesis of its relevance in tail risk quantification, as well as its explanatory analysis and the ability of forecasting by using Time Series models such as ARIMA and GARCH. The tail risk can be defined as the probability that a financial position moves more than two or three standard deviations (in this particular case, to the left of the Profit and Loss distribution) of its mean. Specific studies, such as those performed by Akgiray & Booth (1988) show that returns of financial assets exhibit heavy tails (and, therefore, there is a tail risk), a distribution very different from the traditional normal (*Gaussian*). The reason to study financial risk events or crashes is that the occurrence of a single negative event in a portfolio of assets with heavy-tailed return empirical distribution would cause a significant decrease in the value of the portfolio or the entire economy. Xiong, Idzorek, & Ibbotson (2013) perform an analysis of the crises occurred in 1929, Black Monday (1987) (why Skew Index is precisely created), Asian crisis (1997), the bursting of dotcom bubble (2000), Global financial crisis (2008) and oil crisis (2014). The authors conclude that investors would require a premium to maintain assets with high tail risk.

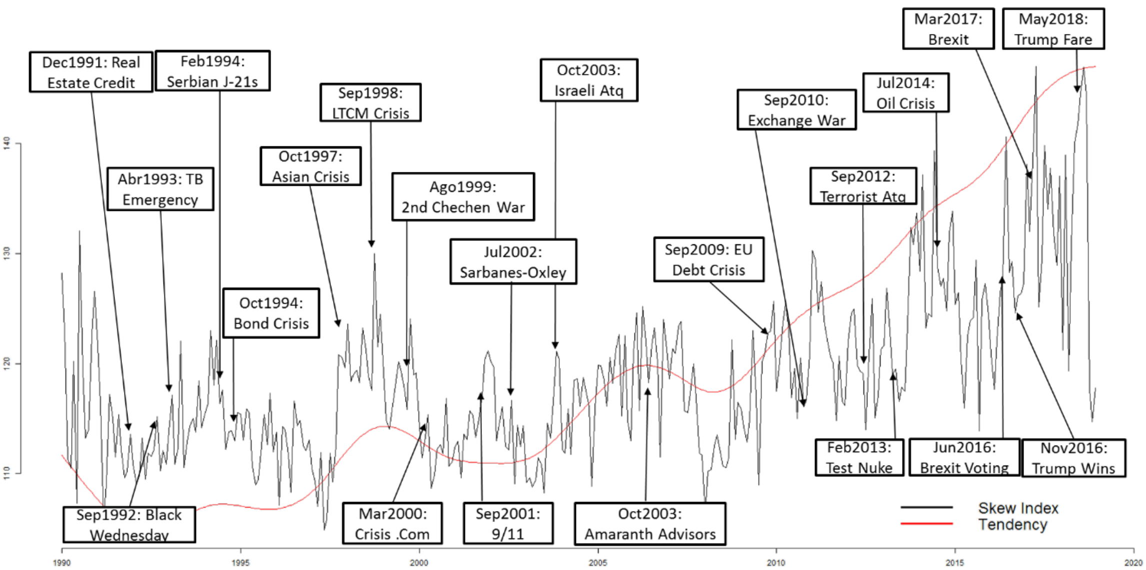

FIGURE 1. SKEW INDEX, TREND AND MAIN ECONOMIC CRISES. SOURCE: THE AUTHOR

These events are taken as milestones in the present analysis. Figure 1 shows the behavior of Skew Index and its tendency, and the main economic crises. This tendency is generated through the Hodrick–Prescott filter,[1] which is widely employed in economic and financial research, and extracts the long-term cyclical and trend components of the Time Series; in general, what it is achieved by using the HP filter, is that the low frequency components are eliminated, whereas high frequency components pass through the Time Series (Serletis, 2006).

According to CBOE sources, the events unleashed in October 1987 put pressure on the main US stock indexes and fell more than 30% in a week, which affected the global economy

[1] The *mFilter* package of R is used (Balcilar, 2018).

before the end of October. The foregoing was the main reason for requesting a study that persue the following controls:

i) the uncontrolled computer automation of purchase and sale of shares,
ii) the increased and uncertainty risk in a market driven by derivative instruments,
iii) illiquidity, boosted by large volumes of sudden stock sales,
iv) the US states own deficit, and
v) the overvaluation of many of the shares in the market (Sornette, 2003).

The main objective of this paper is to analyze the Skew Index, which is employed to respond to investors' need to observe the possibility of extreme falls in the S&P500 returns. According to the studies of the Chicago Board Options Exchange (CBOE), the typical Skew Index's value is around 100 units, where the S&P500 returns (in logarithm) behave normally. However, as this index increases from the value of 100, the loss part of the profit and loss distribution of the S&P500 will increase in size, so the probability of success of these events will increase. The limit zone where the market is said to vary without negative expectations is 150 points.

In the financial literature, there is extensive research on the Volatility Index (VIX). Currently, the VIX calculation method takes into account S&P500's *at-the-money* (ATM) options. In general, it has been found that this index is negatively correlated with the capital market (Gregoriou, 2013), this means that if the S&P500 expected returns present high volatility, values of the VIX Index will be relatively high. However, Black (2006) finds that the asymmetry and leptokurtosis of many hedge funds returns can be smoothed with a long hedge against the spot price of the VIX. That is why Whaley (2009) repeatedly explains that both the VIX and Skew Index are indexes that reflect investors sentiment about the expected volatility in the short term (30 calendar days), which are also classified as "*Fear Index*".

On the other hand, Skew Index is calculated from the prices of the S&P500's Out-of-the-Money (OTM) options. In particular, OTM prices of the put options contain important information on demand of portfolio insurance and, as a result, reflect market volatility (Whaley, 2009). The last, due high and negative correlation between market returns and volatility, which, according to Aboura & Chevallier (2016) measures the tail risk of the S&P500's logarithmic returns' distribution over a 30-day horizon. By then, it is expected as the tail risk of the S&P500 returns increases, the calculated values of Skew Index will also increase. An important difference between VIX and Skew Index is that the Skew Index measures the impact of the distribution asymmetry, VIX approximates the impact of the volatility risk (Aboura & Chevallier, 2016).

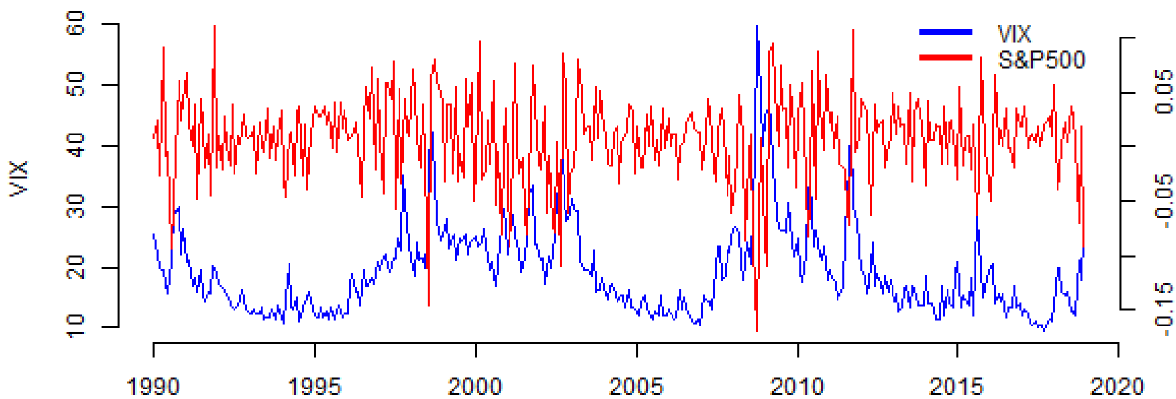

FIGURE 2. VIX AND S&P500'S RETURNS COMPARISON. SOURCE: THE AUTHOR.

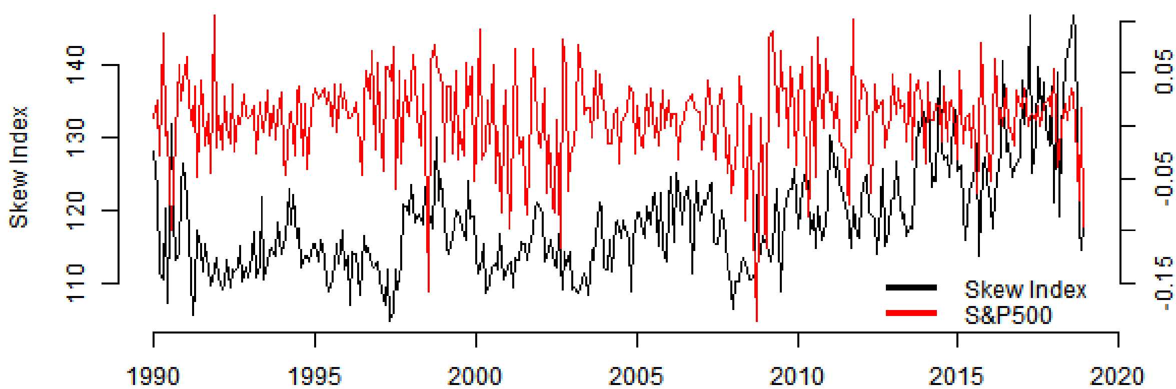

FIGURE 3. SKEW INDEX AND S&P500'S RETURNS COMPARISON. SOURCE: THE AUTHOR

As can be seen in Figure 2, Gregoriou (2013) hypothesis is confirmed, since he argued that the VIX is highly and negative correlated with the returns of the US capital market (using the S&P500 as a proxy). In our case the correlation coefficient between VIX and the S&P500's returns is -0.3782 on monthly basis[2] (see Appendix 1). Likewise, there is a weak relation between the S&P500 returns and Skew Index, showing a correlation coefficient of 0.119. However, the VIX and the Index have in common the quantification of volatility and tail risk

[2] It should be noted that the index created by S&P500 is considered a good proxy of the US capital market in the literature, as is confirmed by (Denis et al. (2003), Gregoriou (2013), and Harris & Gurel (1986), among others

perceived by investors at a given moment in time (Guillaume, 2015), finding a -0.2368 correlation. Additionally, Gregoriou (2013) finds that the correlation between flows of this fear index and the returns of the S&P500 increase in the face of strong market movements as in the global financial crisis, specifically from September 2008 to March 2009. On the other hand, one of the main concerns of market participants and regulators is the systematic risk that occurs in international financial markets during extreme movements of the market (Trabelsi & Naifar, 2017). Therefore, it is necessary to study how to forecast these situations to have better estimates for all the agents that participate in the market.

Another important factor about market's volatility is given by the information surrounding it, the strategies of companies to hide information and legislation that seeks to make disclosure mandatory. In this context, it is relevant to study the date of the approval of the Sarbanes-Oxley Act on July 30, 2002, which, according to studies, could lead to the conclusion that the management of profits has decreased or that companies can keep less information hidden of the business within a new regulatory environment (Hutton, Marcus, & Tehranian, 2009). In Figure 1 can be seen the importance of this event on the behavior of the Skew Index, and how news and regulations can help to shape the economy and its index.

This work is composed in the following way: In section 2, the studies carried out in other financial time series are detailed, including the fear index VIX, which serves as the basis for developing section 3, in which a descriptive analysis with empirical characteristics is presented, based on in Conts (2001) work. Section 4 presents the methodology of the work focused on Time Series Models in order to forecast the Skew Index. Finally, section 5 presents the conclusions of this study.

2. Literature Review

Several studies have been conducted on stock market indexes since the creation of this market, including the recurrently employed Volatility Index (VIX). Skew Index is still in the dark for the academy and investors, so this document is vital to know about its importance for the S&P500 and the market in general.

Kaeck & Alexander (2013) found that sudden jumps in the VIX are more likely during relative calm periods; in addition, the authors traced the days when the occurred jumps coincide with the main political and / or economic events. Precisely, this is one of the facts this research wants to verify for the Skew Index. Along this line, there is also the study carried out by Constantinides on the VIX in 2011 that serves as reference to observe the violations of Skew Index limits at critical periods of the economy. This study uses as milestones some of the historical events mentioned above, which will serve as turning points for the ongoing analysis (Constantinides, Czerwonko, Carsten Jackwerth, & Perrakis, 2011).

Another volatility perspective analysis is developed by Fan et al. (2016), who establish that professionals in the financial industry rely on the sign of volatility risk premium as an indicator of the market expectation of future levels of volatility. They have also found that the increase in high frequency data allows an accurate measurement of the risk premium. On the other hand, Kelly & Jiang (2014) find that tail risk has a great predictive power for the returns of the stock market, aggregated in horizons from one month to five years. In addition, they conclude that tail risk has an explanatory power for the cross section of returns of shares and put options. And Szado (2009) employs comparison between a series of traditional portfolios (composed by different percentages of stocks and bonds), alternative assets (high-yield bonds, hedge funds, etc). This, to analyze the spot VIX and how long volatility exposure may provide protection in downturns and crises. The last is useful for the present analysis since, both Szado's study and the present one are focused on comparative methods of fear indexes, and as its use to determine the expected market's volatility that can provide information on the global economy.

Other studies analyze the VIX Index with various financial tools approaches, for example the American Depository Receipts (ADR), where Esqueda, Luo, & Jackson (2015) have found that VIX can be modeled with a GARCH-M process and be predicted appropriately. According to Mills & Markellos (2008), this is an estimation technique employed when not much is known about the present structure of serial correlation or heteroscedasticity present in the errors of the model used for said forecast. The authors use a (*G)ARCH-in-mean* model to examine the risk premium and to model the shares' return volatility, where the conditional variance is taken into account as a regresor

$$y_t = \beta_0 + \beta_1 x_t + \gamma \sigma_t^2 + \varepsilon_t \tag{1}$$

$$\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2) \tag{2}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{3}$$

McNeil & Frey (2000) found that a conditional approach that models the conditional distribution of asset returns against the current volatility index is more appropriate for estimating the *value-at-risk* (VaR), than an unconditional approach that attempts to estimate the marginal distribution of the process that generates the returns. The modeling system to be used is based on GARCH models. To estimate conditional volatility, using information criteria from Akaike (AIC) and/or Bayesian (BIC) to determine which of the proposed systems to forecast are better suited to the past behavior of Skew Index, as well as the extreme value theory (EVT) to adjust the tail of innovations of a GARCH model proposed.

Regarding the GARCH model, there is evidence of the difficulty to overcome GARCH (1,1) models with more sophisticated systems in the analysis of exchange rates (P. R. Hansen & Lunde, 2005). An idea that emerges from this study is the use of the test for a Superior Prediction Ability (SPA).

In summary, all the studies aimed at establishing the causes of the volatility of the indexes and their influence on the prices of portfolios. Then, the analysis of these causes' aims could improve the predictive capacity of the indexes. In this context, the present study wants to observe what dynamics exist within economic models, when analyzing the S&P500 as a market indexes.

3. Descriptive Analysis of Skew Index and other Indexes.

This section presents the stylized (empirical) facts that arise from the statistical analysis of index variations that serve as a proxy to the capital market (S&P500 and MSCI World Index) and the fear indexes proposed in the literature (VIX and Skew Index). Likewise, other less analyzed fear indexes are presented, in order to try to explain the behavior of the S&P500, 30 days in the future, by means of linear combinations of the fear indexes

3.1 Empirical Characteristics

Following Cont (2001) footsteps, statistical properties of Skew Index are analyzed as a basic step in any empirical study. It is worth noting that this is an important aspect of the study, because it does not exist in the literature review of Skew Index. To begin, it is necessary to analyze the deviation of the average data, since it determines the structure of the tails that are the main point of analysis in the indexes volatility. One way to quantify the deviation from the normal distribution is the kurtosis of the distribution F_T defined as

$$C = \frac{\mu_4}{\sigma^4} - 3 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2} - 3, \tag{4}$$

where μ_4 is the fourth moment with respect to the average, and σ is the standard deviation, and it is understood that, according to Joanes y Gill (1998) a greater kurtosis implies a greater concentration of indexes values near their average as well as the presence of heavy tails, that is to say, that their extreme values are also repeated frequently. Other component taken into account is the asymmetry of said function F_T

$$S = \frac{\mu_3}{\sigma^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{3/2}}, \tag{5}$$

according to Joanes y Gill (1998) it is understood that there is positive asymmetry if the tail to the right of the mean has a greater amount of data than that of its counterpart, indicating there is a lower probability of occurrences of negative returns. Using the Jarque-Bera test as a normality test, which integrates both statistics (kurtosis and asymmetry), formulated below

$$JB = \frac{n-k+1}{6}\left(\left(\frac{C-3}{2}\right)^2 + S^2\right), \tag{6}$$

where n is the number of observations, and k is the number of regressors used. The advantage of using the joint model (having the kurtosis C and the asymmetry S) is the double restriction to agree that said set of data behaves as *Gaussian*. In Econometrics and Time Series Analysis, the Jarque-Bera normality test has been used extensively due to its simplicity and satisfactory results, which has as null hypothesis that the process behaves in a *Gaussian way*; p-values lower than 0.05 would indicate that the series does not behave normally (Górecki, Hörmann, Horváth, & Kokoszka, 2018).

Finally, the Ljung-Box test, which is one of the most common tests for Auto Correlation Function (ACF) inference of the linear Time Series, where we have the ACF of any time series is given by

$$\hat{\rho}_{T,h} = \frac{\sum_{t=1}^{T-h}(y_t - \bar{y})(y_{t+h} - \bar{y})}{\sum_{t=1}^{T-h}(y_t - \bar{y})^2}, h = 0, 1, 2, \ldots, T-1, \quad (7)$$

which serves to determine the Ljung-Box test as follows

$$Q_H = T\,(T + 2) \sum_{h=1}^{H} (T-h)^{-1} \hat{\rho}_{T,h}^2 , \quad (8)$$

where T is the maximum period of time, h a time horizon between data. The Ljung-Box test has an asymptotic distribution to χ^2 for some fixed H (Hassani & Yeganegi, 2019). The null hypothesis of the test is that the time series behaves as white noise[3].

3.2 Descriptive Statistics

By applying the formulas described in the previous section for the monthly returns of S&P500 and MSCI World market indexes, as well as for the observed VIX and Skew Index values; the statisticians would be given by:

TABLE 1. STATISTICS FOR MONTHLY VARIABLES RETURNS

Variable	Mean	Standard Deviation	Kurtosis	Asymmetry	JB P-value	LB P-value
S&P500	0.0067	0.04090	1.32208	-0.61889	0.0000	0.46845
MSCI World Index	0.0045	0.04230	1.46115	-0.65061	0.0000	0.20863
VIX	0.0180	0.20348	6.19039	1.63561	0.0000	0.00357
Skew Index	0.0008	0.04618	2.57181	0.12746	0.0000	0.00000

In Table 1, it can be seen how all monthlyized variables return *means* revolve around zero; besides, it is observed how the Jarque-Bera test p-values imply that none of the indexes (neither market nor fear) have a Gaussian behavior and, with the Ljung-Box test, that both the S&P500 index and MSCI, have absence of autocorrelation. With the above information, it's corroborated the impossibility of making accurate forecasts on these stock indexes, due

[3] According to Brooks (2014), white noise process has a fixed mean and variance but no other structure. For the particular case of the study, one of the structure a Time Series must have to considerate it white noise is the lack of autocorrelations for all lags.

to the completely random movement of these Time Series, and having an expected value (for profitability) of zero, as an econometric wink to the (weak) hypothesis of efficient markets (Fama, 1965; Samuelson, 1965). Otherwise, it could involve cases of arbitrage and allow the prediction of levels of stock indexes. Additionally, it is found that there is a high positive relationship between the monthly returns of the S&P500 index and the MSCI World (part of the *Modern Index Strategy*) with 0.9117 correlation. Both Time Series are representative indexes of the current the global market situation, understanding that the first (S&P500) is entirely focused on the quoting companies in the NYSE or the NASDAQ, and the MSCI World refers to the performance of medium and large capitalization stocks of 23 developed market countries. A positive correlation is also identified between Skew Index and the monthly values of the S&P500, and the MSCI (0.6621 and 0.6366 respectively), thus inferring a relationship in which, while the capitalization of these indexes decreases the outlook of the Investors on the volatility expected in the next 30 days would have to be lower.

3.3 Comparisons with other fear indexes and the market proxy

In order to compare the Skew Index behavior with similar indexes, information is collected from financial fear indexes analyzed by Arak & Mijid (2006), Barone-Adesi, Pisati, & Sala (2018), and Shaikh & Padhi (2015), among others. This way it can be determinate the Skew Index relevance for the S&P500, in addition to the need of having other variables that help calculate the fluctuations of the market proxy, since a single one of these variables does not have enough predictive power on their own.

The indexes based on surveys are:

- *Crash Confidence Index* (CRASH): The information comes from the Yale School of Management website.[4] Despite having information from January-1990 to December-2018, the oldest data until 2001 is semiannual. This index is studied in this document for correlation measures, not for regressions.
- Bull-Bear spread (Bull-Bear). This indicator comes from the American Association of Individual Investors website.[5] Weekly information from July 1988 to December 2018. This index is constructed according to the feelings of market rise *(bullish)* and

[4] https://som.yale.edu/faculty-research-centers/centers-initiatives/international-center-for-finance/data/stock-market-confidence-indices/united-states-stock-market-confidence-indices
[5] https://www.aaii.com/sentimentsurvey

fall *(bearish)*, where the expectation is asked (in percentage) that the prices of the shares will increase and decrease in the next six months, respectively.

The list of fear indexes based on options is composed by:

- CBOE Volatility Index (VIX): Created by the Chicago Board Options Exchange (CBOE) and used by the Federal Reserve of Philadelphia. It has daily information from January 1990 to December 2018.
- CBOE NASDAQ-100 Volatility Index (VXN): It is a measure of market volatility expectations of 30 days for the Nasdaq-100 index, implicit in the price of the options in this index. It has daily information from January 1990 to December 2018, analyzed by Arak & Mijid (2006) and Shaikh & Padhi (2015).
- Implied Volatility Index (IVX): Unlike the VIX, this index allows calculating the volatility of all the stock assets (which participate with options in the market), according to weightings and linear interpolations, regarding the expiration time of the contracts. There is monthly information from June 1995 to December 2018.
- Philadelphia Gold and Silver Index (XAU): As it has been studied by Cohen & Qadan (2010) and Qadan & Yagil (2012), Gold is and always has been the refuge in which fearful investors guard against possible financial crises. There is monthly information from January 1990 to December 2018.

TABLE 2. CORRELATION BETWEEN MONTHLY FEAR INDEXES DATA AND THE S&P500

	S&P500	Skew Index	CRASH	Bull-Bear Spread	VIX	IVX
S&P500	1					
Skew Index	0.7200	1				
CRASH	-0.1107	-0.0681	1			
Bull-Bear Spread	0.0051	0.0683	0.3600	1		
VIX	-0.5041	-0.4340	-0.5039	-0.2621	1	
IVX	0.9776	0.6940	-0.0320	0.0089	-0.5810	1
Gold	-0.3344	-0.1292	-0.3513	-0.2133	0.1062	-0.2914

Table 2 shows the high and positive correlation (0.72) between the S&P500 and Skew Index levels, and the S&P500 with the IVX (0.9776), as well as between the market proxy and its Volatility index (VIX) correlate negatively (-0.5041). This information is used in the next section to verify causality relationships. Likewise, the correlation between monthly logarithmic returns is reviewed. It is started from the fact that there is a large amount of data that cannot be calculated for the CRASH, Bull-Bear Spread nor IVX indexes. The last, precisely because of the log returns calculation nature (which is discussed more thoroughly in the next section). This is why a window of time has been made since 2001 for all the indexes, in order to carry out the study of correlations seen in Table 3. To emphasize, there is a negative correlation (-0.6155) of the log returns of the VIX have a with the S&P500, while the IVX with the S&P500 has a high and positive correlation (0.9594); the other data show no greater relationship.

TABLE 3. CORRELATION BETWEEN MONTHLY LOG-RETURNS OF FEAR INDEXES AND THE S&P500.

	S&P500	Skew Index	CRASH	Bull-Bear Spread	VIX	IVX
S&P500	1					
Skew Index	0.0529	1				
CRASH	0.0695	0.1345	1			
Bull-Bear Spread	0.1971	0.2262	0.1739	1		
VIX	-0.7293	-0.0321	-0.1363	-0.1167	1	
IVX	0.9594	0.0446	0.0711	0.1470	-0.7230	1
Gold	-0.0940	0.0738	0.0814	-0.087	-0.1058	-0.0394

3.4 Approach to the behavior of the market proxy through fear indexes

The above serves as a starting point to know what are the indexes to be used in this study, in addition to the correlation between them. However, the need to give a predictive context makes sense when these fear indexes are based on forecasts, either because they are surveys on market behavior of the coming months, or are calculated with information of the market proxy options purchase/sale with thirty (30) days expiring. It is then that econometric models are analyzed, to determine the significance and forecasting power of Skew Index and the VIX by solitary to the market proxy (S&P500). As well, models that will better amalgamate an interaction between the different fear indexes to try to explain the future behavior of the S&P500. Below are the models used in the study. The statistical results can be found in Appendices 2 and 3 for the model of Ordinary Least Squares (OLS) and Generalized Least Squares (GLS), which have the same values and p-values. In addition, verification with

Generalized Method of Moments (GMM)[6] was performed (Appendices 6 and 7), in which only the value of the intercept changes by decimals, keeping the same information for all other calculations.

Model 1:

$$S\&P500_t = \beta_0 + \beta_1 Skew\ Index_{t-1} + \varepsilon_t, \tag{9}$$

Model 2:

$$S\&P500_t = \beta_0 + \beta_1 VIX_{t-1} + \varepsilon_t, \tag{10}$$

Model 3:

$$S\&P500_t = \beta_0 + \beta_1 Skew\ Index_{t-1} + \beta_2 VIX_{t-1} + \varepsilon_t, \tag{11}$$

Model 4:

$$S\&P500_t = \beta_0 + \beta_1 Skew\ Index_{t-1} + \beta_2 VIX_{t-1} + \beta_3 BullBear\ Spread_{t-1} + \varepsilon_t, \tag{12}$$

Model 5:

$$S\&P500_t = \beta_0 + \beta_1 Skew\ Index_{t-1} + \beta_2 IVX_{t-1} + \varepsilon_t, \tag{13}$$

Model 6:

$$S\&P500_t = \beta_0 + \beta_1 Skew\ Index_{t-1} + \beta_2 IVX_{t-1} + \beta_3 XAU_{t-1} + \varepsilon_t, \tag{14}$$

Model 7:

$$S\&P500_t = \beta_0 + \beta_1 Skew\ Index_{t-1} + \beta_2 VIX_{t-1} + \beta_3 IVX_{t-1} + \beta_4 XAU_{t-1} + \varepsilon_t, \tag{15}$$

What is found in these linear regressions is:

1) Both the Skew Index and the VIX by themselves have explanatory power. We found that p-values of these simple regressions are lower than all levels of uncertainty. It should be noted that residuals associated with these processes do not behave with a Gaussian distribution; the diagnosis made with these models can be doubtful..
2) A unique combination of Skew Index and the VIX as explanatory measures of the S&P500 cannot be used. Although the joint p-value of the model complies with requirements of all values of significance, the VIX itself it just does no, nor does Model 3 errors behave with normal distribution.

[6] According to Brooks (2014), Ordinary Least Squares is a technique to estimate unknown parameters in a linear regression model, assuming that the parameters have the same variance and are not correlated, counting on homoscedastic errors ε_t, while Generalized Least Squares allows an estimation of β_i when the error is of unequal variation (heteroscedastic). On the other hand, The Generalized Moments Method allows the estimation of Models of non-linear rational expectations, which could not be estimated by other methods, besides being employed when problems of endogeneity are encountered, and additionally does not impose restrictions on the data distribution.

3) In of multiple regression models, starting with Skew Index and the *CRASH* index, and adding different variables,[7] in none of the occasions they allowed a significant p-value for the *CRASH* as an explanatory measure of the S&P500.
4) In addition, the studies of the regressions between Skew Index and the *Bull-Bear Spread,* and adding different variables,[8] in none of the occasions the *Bull-Bear Spread* could be recognized as an explanatory measure for the S&P500
5) The IVX and the XAU turn out to be good variables that couple the explanatory power of Skew Index and the VIX jointly, to understand what will happen to the S&P500 a month ahead.

In addition, following the work done by Almeida et al. (2017), the same linear combinations were tested, with logarithmic returns of the S&P500 and other fear indexes, always comparing the market proxy of period *t* with independent variables a period before:

Model 1.1:

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{Skew\ Index_{t-1}}{Skew\ Index_{t-2}}\right) + \varepsilon_t, \tag{16}$$

Model 2.1:

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{VIX_{t-1}}{VIX_{t-2}}\right) + \varepsilon_t, \tag{17}$$

Model 3.1:

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{Skew\ Index_{t-1}}{Skew\ Index_{t-2}}\right) + \beta_2 \ln\left(\frac{VIX_{t-1}}{VIX_{t-2}}\right) + \varepsilon_t, \tag{18}$$

Model 4.1:

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{Skew\ Index_{t-1}}{Skew\ Index_{t-2}}\right) + \beta_2 \ln\left(\frac{VIX_{t-1}}{VIX_{t-2}}\right) + \beta_3 \ln\left(\frac{BullBearSpread_{t-1}}{BullBearSpread_{t-2}}\right) + \varepsilon_t, \tag{19}$$

Model 5.1:

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{Skew\ Index_{t-1}}{Skew\ Index_{t-2}}\right) + \beta_2 \ln\left(\frac{VIX_{t-1}}{VIX_{t-2}}\right) + \varepsilon_t, \tag{20}$$

Model 6.1:

[7] Many more combinations of variables were tested than those included in this document. They will be available at the request of the reader.
[8] More combinations were performed with explanatory variables looking in a futile way for a joint explanation of these to the S&P500. They are available if requested by the reader.

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{Skew\ Index_{t-1}}{Skew\ Index_{t-2}}\right) + \beta_2 \ln\left(\frac{IVX_{t-1}}{IVX_{t-2}}\right) + \beta_3 \ln\left(\frac{XAU_{t-1}}{XAU_{t-2}}\right) + \varepsilon_t, \quad (21)$$

Model 7.1:

$$\ln\left(\frac{S\&P500_t}{S\&P500_{t-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{Skew\ Index_{t-1}}{Skew\ Index_{t-2}}\right) + \beta_2 \ln\left(\frac{VIX_{t-1}}{VIX_{t-2}}\right) + \beta_3 \ln\left(\frac{IVX_{t-1}}{IVX_{t-2}}\right) + \beta_4 \ln\left(\frac{XAU_{t-1}}{XAU_{t-2}}\right) + \varepsilon_t, \quad (22)$$

The findings in these linear regressions can be found in Appendices 4 and 5 for OLS and GLS, in addition to Appendices 8 and 9, where the same values and statistical significance are found; the conclusions on the subject are:

1) Both, Ordinary Least Squares and Generalized Least Squares methods, have the same estimators for β_i in all cases of all models with same p-values.
2) There is the impossibility of estimating the Model 4.1 with the Generalized Momentary Method, nor calculating the log returns $R_{BBS} = \ln\left(\frac{bull\ bear\ spread_t}{bull\ bear\ spread_{t-1}}\right)$, nor with the returns in a simple way $R_{BBS} = \frac{BBS_{t-1} * BBS_t}{BBS_t}$,[9] by which the algorithm can't be completed. OLS and GLS models can be performed by re-dimensioning the series, because there is a large amount of non-existent data, due to the nature of the calculation of logarithmic returns.
3) Neither the logarithmic returns of Skew Index nor VIX alone have explanatory power. The p-values of these simple regressions are much higher than any level of significance, both for the individual test and for the joint test.
4) No possible combination with the fear indexes contemplated in this study yields p-values that are below the levels of significance required to be able to accept explanatory power of the independent variables towards the logarithmic returns of the S&P500.[10]

[9] This implies that any combination that includes the existence of the logarithmic return of the *bull-bear spread* can't be realized.

[10] Many more combinations of variables were tested than those included in this document. They will be available at reader's request.

There is statistical information corroborating that Skew Index levels does have explanatory power over the S&P500 levels. Therefore, it has relevance as a market predictor, which increases the importance of a study on the possibility to predict the movements of this stock index, to try take shock measures against future movements that influence fear in investors, which is precisely the subject of the following section.

4. Forecast Methodology

One way to approach the problem of correctly modeling the prediction system for the Skew Index time series is to take the same path as Tully & Lucey (2007), where relations are analyzed in two periods of crisis for stock markets, examining a period of a few years around a collapse. In their work, the authors analyze around two global events: October 1987's crash (reason why Skew Index exists), as well as the bullish peak of March 2001, where they use gold as a prediction tool, applying APGARCH systems. The authors incorporate Power GARCH models with asymmetry, which allows to incorporate both leverage and the Taylor effect, which explains Ghalanos (2018), who observed that the autocorrelation of the sample of the absolute returns was generally greater than that of the squared yields. Tully & Lucey (2007) find that using cash and futures data over a prolonged period, they confirm that the US dollar is the main macroeconomic variable that influences gold prices, so we can bring up indexes that can be used in this study, for which we include gold (XAU).

Additionally, this study presents a comparative analysis between the Skew Index and the real value of the standard deviation of the S&P500's returns. The idea is to contrast Fan, Imerman, & Dai (2016) conclusions, who question the equality between the volatility of the risk premium to a systematic price bias between the volatility carried out ex post and the expected ex-ante volatility implicit in options..

4.1 ARIMA Modeling

Given the relevance of Skew Index for the US capital market, this chapter seeks to model the time series in such a way that its future behavior can be forecasted correctly, including values of uncertainty about its stochastic movement.

As a previous step to the development of the models, stationarity for the studied time series is tested. One of the best ways to verify that this is true is by performing the Augmented Dickey-Fuller test (ADF), where the null hypothesis (H_0) is that the autoregressive model has a unit root. Therefore, is needed is that the time series doesn't have unit root and the ADF test would reject its H_0. According to the above, and observing the results of Table 4, it is found that both the S&P500 Index and the MSCI World Index have a unit root, as does the VIX; Skew Index does not meet this condition, showing that it behaves stationary.

TABLE 4. AUGMENTED DICKEY-FULLER TEST ON VARIABLE LEVELS

Variable	ADF	P-Value
S&P500	-1.7677	0.6748
MSCI World Index	-2.8932	0.1999
VIX	-3.0149	0.1485
Skew Index	-4.1723	0.01

*Ho: Variable has a unit root.

The next step is to verify that the temporal variables do not behave as white noise, since a series that is not auto correlated cannot be studied under the ARIMA models and, therefore, could not be predicted with this approach. Following the Ljung-Box test, it can be determined if the guess has been made correctly. The null hypothesis of this test is the absence of autocorrelation. The statistical test is given by

$$Q^* = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k}, \qquad (23)$$

where *n* is the simple size, $\hat{\rho}_k^2$ is the sample lag *k* autocorrelation, and *h* is in number of lags being tested. Given the p-values, it is interpreted that all the study variables behave in a dependent manner. When differentiating these monthlyized variables, it can be seen how their p-values pass the null hypothesis that there is no autocorrelation, except for Skew Index, for which ARIMA models can be used to predict the future behavior of said Skew Index (See Table 5).

TABLE 5. LJUNG-BOX TEST. LACK OF AUTO CORRELATION ON DIFFERENTIATED VARIABLES

Variable	Ljung-Box	P-value
MSCI World Index	2.14	0.14
S&P500	0.26	0.61
VIX	1.21	0.27
Skew Index	62.81	0.00

*Ho: Absence of autocorrelation.

Since Skew Index is taken as an univariate information series, as explained by Diebold (2017), The next step is to test the Models, where the explanatory components that could be found in Skew Index are verified.

- Model 1: It assumes exclusively the existence of a linear trend,
- Model 2: There is a quadratic behavior in the trend,
- Model 3: A system without trend,
- Model 4: A transformation of the time series (Skew Index) is done logarithmically, which, according to Diebold (2018), is effective for systems that turn out to be nonlinear in a pure way, but with this type of conversion. In addition to this, we work with seasonality and cubic trends component systems, and,
- Model 5: which is the logarithmically Skew Index Time Series, and with seasonal and trend components.

The ARIMA verification is done with the computer software for each described model, which yields the following results, where both the order of their terms (p, d, q) and the (AIC[11] and BIC[12]) information criteria are interested. According to Diebold (2018), these criteria's nature is the penalization of waste within the sample, in order to have a more accurate estimate of off-sample forecasts. The statistics are given by:

$$AIC = \frac{e^{\frac{2K}{T}} \sum_{t=1}^{T} e_t^2}{T}, \quad (24)$$

$$BIC = \frac{T^{\frac{K}{T}} \sum_{t=1}^{T} e_t^2}{T}, \quad (25)$$

where K are the degrees of freedom, referenced as the number of variables that the Model has to the right side of the equation of the ARIMA Model proposed by the researcher, including the intercept.

TABLE 6. ARIMA MODELS USED WITH INFORMATION CRITERIA FOR SKEW INDEX LEVELS

Model	SARIMA	AIC	BIC

[11] According to Diebold (2018), the Akaike information criterion is an estimate of the variance of the out-of-sample forecast error, penalizing the degrees of freedom. Together with the BIC, it is employed to select between forecast models proposed by the researcher.
[12] The Bayesian or Schwarz criterion information is an alternative to the AIC with the same interpretation, but with a more severe penalty on the degrees of freedom. Diebold (2018)

1[13]	(1,0,1)(0,0,1)12	2010.46	2033.43
2	(1,0,1)	2002.16	2025.14
3	(1,1,2)(0,0,1)12	2008.32	2027.45
4	(1,0,1)	-1247.51	-1178.59
5	(1,0,1)	-1240.72	-1179.46

* The lower value of AIC and BIC indicate the best Model

Both, with the model with linear trend (Model 1) and with the quadratic trend (Model 2) it shows that it is an ARIMA (1,0,1) (0,0,1) 12, while running a R[14]Auto ARIMA it is found that the best adjusted model is (1,1,2) (0,0,1) 12. The information criteria show that Model Number 1 is discarded, while Model 2 is better than Model 3 according to Akaike and Bayesian criteria.

A control is carried out on the five (5) Models residuals, following the Ljung-Box test it is possible to determine if conjectures and approaches have been made correctly. The null hypothesis establishes that the residuals data distribute with no correlation what so ever. Given the p-values, it is interpreted that there is a 92.05% and 77.67% (respectively) probability that the residues of Models 4 and 5 behave randomly, or that they behave as white noise; much higher than the 0.05 necessary to reject the null hypothesis of dependence on these residues (necessary to approve the models studied).

TABLE 7. TEST LJUNG-BOX. LACK OF AUTO CORRELATION ON IMPLEMENTED MODEL'S RESIDUALS

Model	Q*	P-Value
1	28.064	0.1079
2	33.191	0.0442
3	27.305	0.0978
4	0.0100	0.9205
5	0.0804	0.7767

*Ho: Absence of autocorrelation.

[13] The composition of Model 1: (1,0,1) (0,0,1) 12 shows both its autoregressive and immediate moving average components, as well as the temporary ones. For this specific case the equation that would accompany this Model would be given by:

$$SkewIndex_t = Tendency_t + \phi_1 SkewIndex_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_{12} \varepsilon_{t-12}$$

[14] From the R package *forecast*, auto.arima returns the best ARIMA model according to AIC, AICc or BIC, using sum of squares conditional to find initial values and as maximum likelihood adjustment method. The information supplied by this procedure are the regular orders of the AR, I and MA (p, d, q), the seasonal orders of the AR, I and MA (P, D, Q), and the periodicity of the series (s). (Hyndman et al., 2019)

Often, the same Model is selected according to information criteria, indistinctly if a comparison is made with the Akaike, Schwarz or others.[15] When this is not the case, despite the theoretical property of asymptotic efficacy of AIC, Diebold (2018) recommends the use of the most parsimonious model selected by the SIC, *ceteris paribus*. Both Models 4 and 5 have the lowest Akaike and Schwarz information criteria with very small differences, in addition to residues they behave as white noise. Taking into account the principle of parsimony, we choose to carry out the forecast with the time series ARMA (1, 1) for the Skew Index logarithmically, and with a seasonal and trend component, called Model 5, from now on:

$$log(Skew)_t = log(Skew)_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} + Tendecy_t + Seasonal_t$$

$$log(Skew)_t = log(Skew)_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} + (\mu_1 + \mu_2 t) + \sum_{i=1}^{s} \gamma_i D_{it}, \quad (26)$$

where

$$S_t = \begin{cases} \gamma_1 & ssi \quad t = ene \\ \gamma_2 & ssi \quad t = feb \\ \vdots & \quad \vdots \\ \gamma_{12} & ssi \quad t = dic \end{cases} \quad (27)$$

4.2 ARIMA and GARCH Modeling

Despite the results found in the previous point, it was decided to try to work with a logarithmic return model instead of the previous level system of the Skew Index. It is done precisely to capture the volatility jumps in the US stock market, in addition to trying to find a more robust model, according to the financial theory of stock prices in the stock market.

In financial theory, it may be more important than the realization of events, how they influence the expected return, which is what the investor really looks for. This is why in this section the research is turned towards the Skew Index monthly returns; try to model it, finding how its mean and variance behave. The first is to understand that there are two measures of

[15] Claeskens & Hjort (2008) expose a number of information criteria, such as the Deviation Information Criterion (DIC), Focused Information Criterion (FIC), Takeuchi Information Criteria (TIC) or Hannan-Quinn Information Criteria (among others). The most used in econometric studies and in statistical packages are Akaike and Schwarz, for which they will only be the last two related in this document.

return for a given period, which are the simple and logarithmic (returns), formula 28 and 29 respectively,

$$R_{[t,t-1]} = R_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \tag{28}$$

$$R_{[t,t-1]} = R_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{29}$$

where, P is the stock (or, in this case, Index) price, comparing the price of the current period *t* with an immediately preceding period $(t-1)$ or, in a generalized way, any previous period $(t-h)$, as long as the horizon *h* does not exceed the total data's length. According to Panna (2017), one of the main reasons why the logarithmic return is used instead of the simple return is that, although adding numbers close to zero does not represent a problem, multiplying numbers close to zero can cause an arithmetic overflow.

The next step is, through the auto.arima process (see footnote 15), to find the best candidates to work with the auto regressive and moving average models, depending on pre-established parameters. In Model 6 (following the nomenclature of the previous section), it is found that, when a linear trend component is added, the best model is the MA (4), which can be represented by in equation 30,

$$R_t = Tendecy_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \theta_4 \varepsilon_{t-4}, \tag{30}$$

whereas, without adding trend models, the best possible result is a SARIMA (1, 0, 2) (0, 0, 1) 12, shown in equation 31,

$$R_t = \phi_1 R_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_{12} \varepsilon_{t-12}. \tag{31}$$

TABLE 8. ARIMA MODELS USED WITH INFORMATION CRITERIA FOR SKEW INDEX LOG RETURNS

Model	ARIMA	AIC	BIC
6	(0,0,4)	-1236.97	-1213.87
7	(1,0,2)(0,0,1)12	-1236.16	-1216.91

* The lower value of AIC and BIC indicate the best Model

In Table 7, both models are explained, along with their Akaike and Schwarz values.[16] Model 6 is chosen to perform the study, due to three main reasons:

1) there is no significant difference between the two models according to information criteria,

2) for parsimony, we choose the model with fewer explanatory variables,

3) the rugarch package of R, in its most recent version (version 1.4-1, 2019),[17] does not have the possibility of adding SARIMA Models, but it does have ARMA with external regressors.

Engle (1982) proposes the verification of conditional heteroscedasticity, in which the variance of the term of the innovation of time *t,* is a function that has as parameters the previous periods error terms. In the vars R package[18] it is found the function "*arch.test*", based on Engle´s principle, verifies that a given model, in this case the residuals of Model MA (4), meets the null hypothesis that no ARCH effects are found. The Model MA (4) does not comply with this proposal, counting on a p-value of 4.253e-07, lower than all the confidence measurements, for which reason it is decided to propose ARCH Models with different parameters.

In an iterative manner, models sGARCH, eGARCH, gjrGARCH and iGARCH are proposed and tested, with different AR components and conditional heteroscedasticity. These Models described below, as explained by Ali (2013); Casas & Cepeda (2008; Glosten, Jagannathan, & Runkle (1993):

[16] More combinations were performed with different quadratic and cubic trends, in addition to dummies to model the seasonal component, which did not show a better performance according to information criteria than models 6 and 7. The results of these study are available at request of the reader.

[17] According to the description of the package by Ghalanos (2019).

[18] For more information, refer to Pfaff (2008)

Exponential GARCH (eGARCH):

$$\varepsilon_t = \sigma_t z_t; ln\sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i ln\sigma_{t-j}^2, \qquad (32)$$

gjrGARCH:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_t^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-j}^2 + \sum_{j=1}^{p} \gamma_i I_{t-1} \varepsilon_{t-1}^2, \qquad (33)$$

where

$$I_{t-i} = \begin{cases} 1 & si\ \varepsilon_{t-1} < 0 \\ 0 & si\ \varepsilon_{t-1} \geq 0' \end{cases} \qquad (34)$$

Integrated GARCH (iGARCH):

$$\varepsilon_t = \sigma_t z_t; \sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{j} \beta_i \sigma_{t-j}^2, \qquad (35)$$

Where the coefficients sum[19] is restricted to 1, main difference with the simple GARCH. This is why Models iGARCH are strongly stationary without being weakly stationary (Nelson, 1990). What was found in this iterations process is that the model that best behaves is the eGARCH (1,3) with innovations distributed as *t-student*[20] for the variance, and a trend Model and MA (4) for its mean (denominated as *Model 8* from now on). Models with lower ARCH order did not show that the conditional heteroscedasticity of the Model was fulfilled (performing Engle test after running the Model). Larger orders, such as GARCH(1,4) and higher, have large innovations standard deviations, which destabilizes said model, implying wide confidence intervals, resulting in uncertain information about the future occurrence of the Skew Index logarithmic returns, nor of its possible levels. The results of the different GARCH Models (1, 3) with variation of the way innovations are distributed are shown in Appendix 10, where it can also be seen the results of forecasts within sample, which are explained in the chapter on forecasts.

[19] $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{j} \beta_i = 1$

[20] The results of these iterations are available at the request of the reader.

4.3 **Forecasts**

As a first measure, a forecast is made within the sample, dividing the time series into two equal parts [21] predicting one period forward at a time. For this, a time window is used to forecast base with Model 8 for the logarithmic returns of the Skew Index (removing the seasonality and trend components of Model 5). The goal is to achieve the highest *success rate*, such that the value observed from the Skew Index, be within the confidence interval of each of the given forecasts.

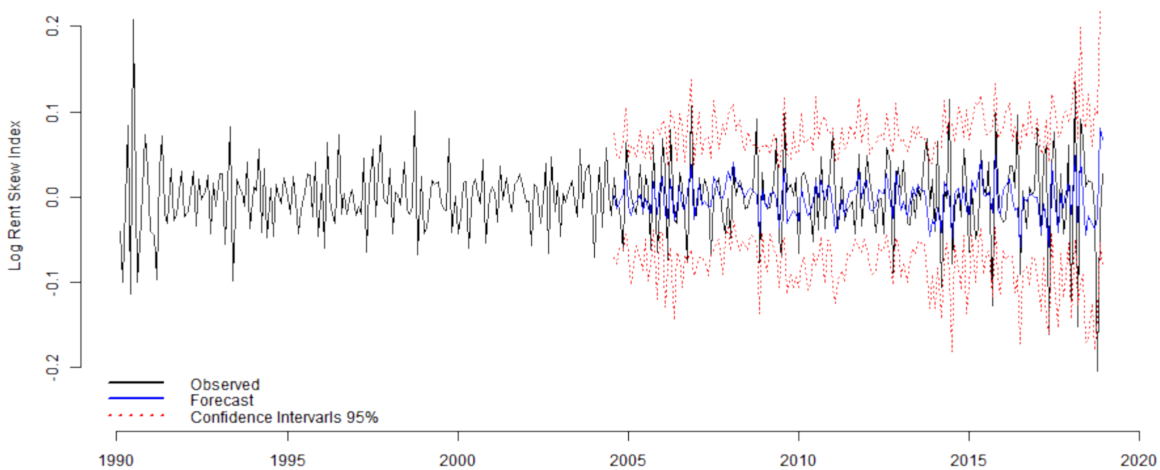

FIGURA 4. SKEW INDEX MONTHLY LOG RETURNS IN-SAMPLE FORECAST. SOURCE: THE AUTHOR

In Figure 4, the forecast within the sample can be observed, using the first half of the series of returns of the Skew Index as a basis for the projection of the returns of the same index. Taking into account that the logarithmic profitability of the Skew Index is given by

$$R_t = \ln\left(\frac{SkewIndex_t}{SkewIndex_{t-1}}\right), \tag{36}$$

that there is the Skew Index level data, and with the previous returns' forecast, it can be found the predicted in-sample levels for the Skew Index in period *t* of Formula 36, such that

$$e^{R_t} = \frac{SkewIndex_t}{SkewIndex_{t-1}}$$

$$SkewIndex_t = e^{R_t} * SkewIndex_{t-1}, \tag{37}$$

so it is obtain the data to construct Figure 5.

[21] By this procedure, one period at a time can be predicted with a time window of half the full length of the time series. For the Skew Index, analyzed from January 1990 to December 2018, there are 348 monthly data. Each window will have 174 monthly data, while for the monthly return model we would have 347 data, so the time window is asymmetric, counting on the first half (which is the basis for the study) with 174 data, while the segment to be forecast has 173.

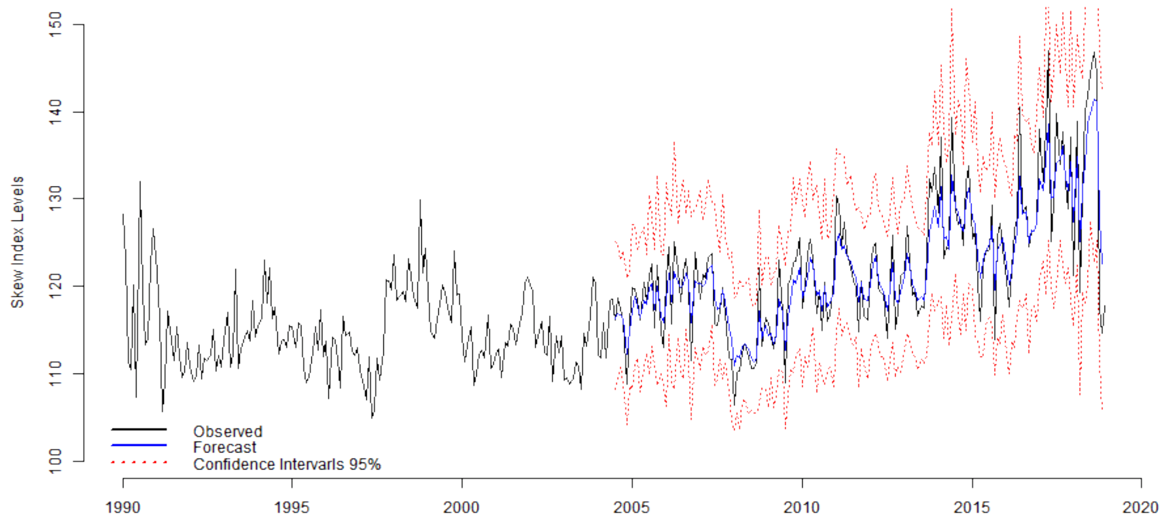

FIGURE 5. SKEW INDEX MONTHLY LEVELS IN-SAMPLE FORECAST. SOURCE: THE AUTHOR

It is found that, using a time window that increases in a period each time it's approach the last observed value, setting the initial moment as January 1990 (Skew Index origin date), a success rate of 93.0636% is obtained, mean absolute error of 0.03395 and mean square error of 0.00156.[22] This prediction model is carried out within the sample with different GARCH models, as well as models of *normal distribution, t-Student* and *asymmetric t-Student* for residues, which can be observed in Appendix 10. No large differences between GARCH models proposed for the study are found. Nevertheless, consistency is observed in the eGARCH, which passes all the safety filters, such as Ljung-Box Test in Standardized Residuals, Nyblom statistics test of stability, and Engle Lagrange Multiplier for asymmetric effects values.[23] The las information its why eGARCH model is chosen to forecast the performance of the Skew Index returns, to later find the forecasted levels of this financial fear index.

Next, the self-predictive capacity of the Skew Index is tested against the volatility analyzed in reference to the historical milestones of the economic crashes, with Model 8 (MA (4) and eGARCH (1, 3)). This is contrasted with a random walk model with the aim of establishing

[22] As it is explained by Poon & Granger (2003), both prediction of first and second moments of a time series for financial markets (and in general) are equally important, but it is even more corroborating that the models that are trying to identify the mean and the variance of these systems are successful. These same authors propose the MAE and MSE to evaluate the predictive power of different GARCH models within the sample, in different time horizons.

[23] It allows the verification of the proposed model's conditional auto-regressive heteroscedasticity, with the null hypothesis that there are no ARCH effects. Likewise, it is recommended to carry out this process after a first "correction" of ARCH effects, to determine that it has been done thoroughly. (Catani & Ahlgren, 2017)

to what extent the Skew Index can contribute in the prediction of market volatility.

TABLE 9. 12 MONTH LEVEL FORECAST WITH 95% INTERVAL CONFIDENCE

	Model 8			Random Walk		
	5%	Forecast	95%	5%	Forecast	95%
jan-19	111.481	121.744	132.951	107.721	118.038	128.355
feb-19	112.012	125.061	139.629	105.498	118.209	130.921
mar-19	113.348	126.653	141.520	104.542	118.333	132.123
apr-19	116.361	127.218	139.087	104.104	118.421	132.738
may-19	113.960	127.218	142.018	103.903	118.485	133.068
jun-19	114.567	127.218	141.265	103.813	118.531	133.250
jul-19	116.203	127.218	139.277	103.776	118.564	133.353
ago-19	114.120	127.218	141.819	103.764	118.588	133.412
sep-19	115.109	127.218	140.601	103.762	118.605	133.448
oct-19	116.054	127.218	139.456	103.765	118.618	133.470
nov-19	114.372	127.218	141.506	103.769	118.627	133.484
dec-19	115.510	127.218	140.112	103.773	118.633	133.493

Testing Model 8, Skew Index values are forecasted, as well as values to any degree of uncertainty that the researcher wants to incorporate. Table 8 shows these forecast points, as well as the 95% confidence intervals, both for Model 8 and for *Random Walk*. Graphically it can be observed in Appendices 11 and 12, where it is found, for both models respectively, the 95% CI[24], according to the R *forecast* package standards. (Hyndman et al., 2019).

Taking into account the difference between extreme values of these two models (Figure 6), it's seen how the random walk model's variance is bigger than Model 8's. In Appendix 13, it can be seen that they fallow different trajectories, where the Random Walk diverges quickly to its average, while the Model 8 has incorporated the volatility dynamics, its own information four (4) months ago, and its innovations.

[24] Confidence Intervals

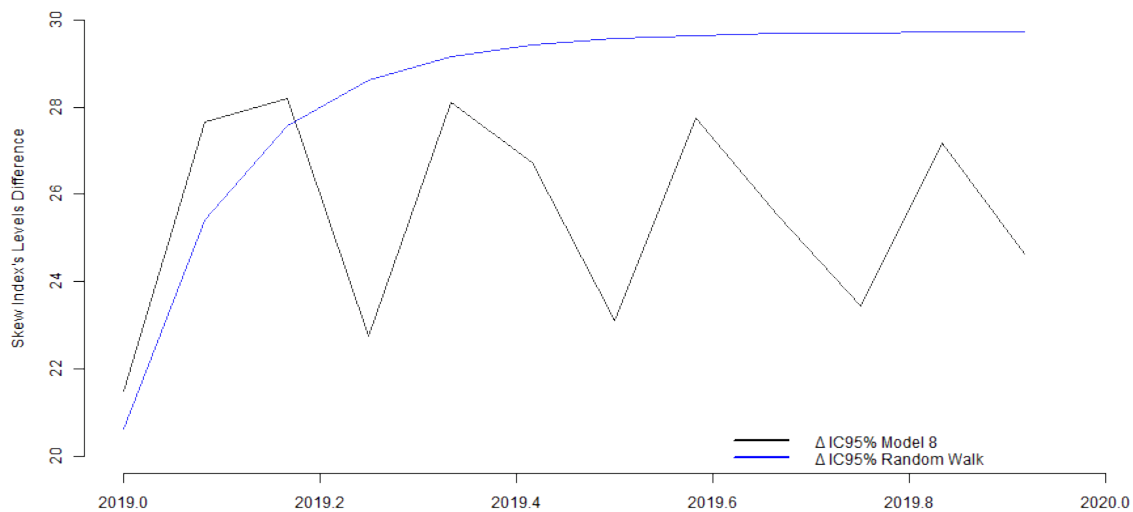

FIGURE 6. FORECASTS' DENSITY DIFFERENCES COMPARISON. SOURCE: THE AUTHOR.

5. Conclusions

As expected, there is a high positive correlation between the S&P500 and the MSCI World Indexes, both being global market representative indexes and its current situation. The S&P500 is taken from stock listed in the NYSE and NASDAQ, and MSCI World Index from the *Modern Index Strategy*.

What is interesting is that there is a positive correlation between the Skew Index and these two indexes. This could infer that, as capitalization in these market indexes increases, the investors' perspective on the volatility expected in the next 30 days would have to be bigger. Although it should be remembered the spurious of *cum hoc ergo propter hoc*, "the correlation does not indicate a direct causal relation, but a common dependence on a third variable" (Aldrich, 1995, p. 364). Therefore, causality studies are carried out, with Ordinary Least Squares, Generalized Least Squares and Generalized Method of Moments regression models. It is demonstrated that the Skew Index one previous period level does work as an explanatory measure of the S&P500, which is a good proxy of the market, the latter according to Denis, McConnell, Ovtchinnikov, & Yu (2003), Gregoriou (2013) y Harris & Gurel (1986). The last is only true in two situation:

1) If the Skew Index works by itself, doubting on the model diagnosis since the residuals behavior does not have a Gaussian distribution.

2) If the Skew Index works in simultaneous with variables as the IVX (Implicit Volatility Index), XAU (Gold) and, importantly, with the VIX (Volatility Index).

This is why it is suggested to use different approaches, such as taking exclusively data from month-end, instead of daily data, as well as ARIMA and GARCH models to have a better approximation to the Skew Index forecast. It is found a model that predicts the values taken by the index with 5% uncertainty in a time window of twelve (12) months. Regrettably, the monthlyized VIX behaves as *White Noise* in its differential mode. The models proposed by the research, corroborated using appropriate statistical packages, was precisely where the VIX should be integrated once, which is why you cannot perform Auto Regressive Vector Models, which comply with ARIMA requirements.

For the next study, it is proposed to perform Neural Networks Model or *Machine Learning*, which do not require the *stationarity* nor *White Noise* principles. One of the main reasons why it is desired to use more than one explanatory variable is because, according to Kelly & Jiang (2014) estimates based on univariate time series of aggregate market returns are unable to accurately track the risk of conditional tail. According to Gregoriou (2013), the *CBOE VIX Tail Hedge Index*, which is calculated using VIX call options to protect portfolios of stocks against tail risks, yields less than the S&P500 during periods of calm (low volatility). This is due to the protection premium paid, which would also be observed if we made the same coverage with the Skew Index; the latter can be the basis for future research, predicting both the values of the VIX, Skew Index and the *VIX Tail Hedge Index*. It is also imperative to analyze what other factors influence the assets prices, for example, analysis of the power law and the real effects of the uncertainty shocks that represent latent channels through which the tail risk at the level can influence the prices of asset (Kelly & Jiang, 2014).

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Appendices

	Skew Index	VIX	S&P500	MSCI-W	Kansas	FTSE 100	FTSE 250	Kospi
Skew Index	1							
VIX	-0.2368	1						
S&P500	0.119	-0.3782	1					
MSCI-W	0.11	-0.3871	0.9117	1				
Kansas	-0.0476	-0.0524	0.0171	0.007	1			
FTSE100	0.0528	-0.3301	0.7663	0.7915	0.0112	1		
FTSE250	0.033	-0.3296	0.6752	0.7357	0.0126	0.8293	1	
Kospi	0.0648	-0.1422	0.4311	0.5064	-0.0585	0.4635	0.4749	1
Nikkei	0.1044	-0.3195	0.5186	0.6849	3.00E-04	0.4595	0.5297	0.4429

APPENDIX 1. MONTHLY CORRELATION MATRIX – FEAR INDEX LEVELS VS STOCK MARKET RETURNS

Panel A: Ordinary Least Squares - Simple | Dependent Variable: S&P500 Monthly Levels				
	Model 1		Model 2	
	Value	P-Value	Value	P-Value
Intercept	-5166.985	<2e-16	1489.116	<2e-16
Skew Index	53.7930	<2e-16		
VIX			-14.171	0.0012
Degrees of Freedom	345		345	
Adjusted R^2	0.4470		0.0272	
F Test	280.700	0.0000	10.670	0.0012
Jarque Bera Test[25] Residuals	14.3788	0.0008	10.3364	0.0057

APPENDIX 2. ORDINARY LEAST SQUARE RESULTS - SIMPLE REGRESSION - MONTHLY VARIABLE LEVELS

[25] Jarque Bera Test: normality test. It is a joint test proposed by Jarque & Bera (1980), with null hypothesis that a data series has kurtosis = 3 and asymmetry = 0 is evaluated together, which is statistical evidence of a Gaussian behavior.

Panel B: Ordinary Least Squares - Multiple | Dependent Variable: S&P500 Monthly Levels										
	Model 3		Model 4		Model 5		Model 6		Model 7	
	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value
Intercept	-5098.281	<2e-16	-5147.661	<2e-16	-779.979	<2e-16	-741.202	0.000	-868.545	0.000
Skew Index	53.455	<2e-16	53.699	<2e-16	5.370	<2e-16	6.074	0.000	6.241	0.000
VIX	-1.479	0.661	-0.899	0.795					3.249	0.001
IVX					2.3648	<2e-16	2.3116	<2E-16	2.3665	<2e-16
XAU							-0.817	0.000	-0.755	0.000
Bull Bear Spread			108.3503	0.4309						
Degrees of Freedom	347		347		282		282		282	
Adjusted R^2	0.4457		0.4451		0.9484		0.9530		0.9547	
F Test	140.1	0.0	93.5	0.0	2583.0	0.0	1901.0	0.0	1481.0	0.0
JBT[26] Residuals	13.5574	0.0011	14.5508	0.0007	2.4642	0.2917	0.0369	0.9817	0.5136	0.7735

APPENDIX 3. ORDINARY LEAST SQUARE RESULTS - MULTIPLE REGRESSION - MONTHLY VARIABLE LEVELS

[26] Jarque Bera Test: Normality test

Panel A: Ordinary Least Squares - Simple | Dependent Variable: S&P500 Monthly Log Returns				
	Model 1.1		Model 2.1	
	Value	P-Value	Value	P-Value
Intercept	0.0058	0.0091	0.0058	0.0090
Skew Index	-0.0648	0.1784	-0.0162	0.1781
VIX				
Degrees of Freedom	344		344	
Adjusted R^2	0.0366		0.0024	
F Test	1.8180	0.1784	1.8210	0.1781
Jarque Bera Test Residuals	82.4665	0.0000	64.9662	0.0000

APPENDIX 4. ORDINARY LEAST SQUARE RESULTS - SIMPLE REGRESSION - MONTHLY VARIABLE LOG RETURNS

Panel B: Ordinary Least Squares – Multiple | Dependent Variable: S&P500 Monthly Log Returns										
	Model 3.1		Model 4.1		Model 5.1		Model 6.1		Model 7.1	
	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value
Intercept	0.0058	0.0092	0.0085	0.0021	0.0050	0.0540	0.0049	0.0583	0.0051	0.0513
Skew Index	-0.0703	0.1448	-0.0472	0.4701	-0.0714	0.2170	-0.0709	0.2209	-0.0721	0.2134
VIX	-0.0176	0.1446	0.0088	0.5934					-0.0124	0.4717
IVX					0.0741	0.2090	0.0829	0.1729	0.0509	0.4991
XAU							-0.0151	0.5434	-0.0163	0.5146
Bull Bear Spread			0.0025	0.3617						
Degrees of Freedom	343		158		278		277		276	
Adjusted R^2	0.0057		-0.0102		0.0022		-0.0001		-0.0018	
F Test	1.9810	0.1395	0.4594	0.7110	1.3030	0.2735	0.9899	0.3980	0.8710	0.4817
JBT[27] Residuals	61.9603	0.0000	5.1820	0.0749	54.2293	0.0000	57.1428	0.0000	52.2762	0.0000

APPENDIX 5. ORDINARY LEAST SQUARE RESULTS - MULTIPLE REGRESSION - MONTHLY VARIABLE LOG RETURNS

[27] Jarque Bera Test: Normality test

Panel A: Generalized Method of Moments - Simple | Dependent Variable: S&P500 Monthly Levels				
	Model 1		Model 2	
	Value	P-Value	Value	P-Value
Intercept	-5167.00	0.0000	1489.12	0.6452
Skew Index	53.7930	0.0000		
VIX			-14.1710	0.8869
J Test[28]	0.0000		0.0000	
Jarque Bera Test Residuals	14.3788	0.0008	10.3364	0.0057

APPENDIX 6. GENERALIZED METHOD OF MOMENTS RESULTS - SIMPLE REGRESSION - MONTHLY VARIABLE LEVELS

Panel B: Generalized Method of Moments - Multiple | Dependent Variable: S&P500 Monthly Levels										
	Model 3		Model 4		Model 5		Model 6		Model 7	
	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value
Intercept	-5098.30	0.0000	-5147.70	0.0000	-779.98	0.0033	-741.20	0.0006	-868.54	0.0001
Skew Index	53.4550	0.0000	53.6990	0.0000	5.3700	0.0297	6.0742	0.0033	6.2412	0.0020
VIX	-1.4794	0.7328	-0.8991	0.8390					3.2485	0.1314
IVX					2.3648	0.0000	2.3116	0.0000	2.3665	0.0000
XAU							-0.8168	0.0088	-0.7552	0.0155
Bull Bear Spread			108.350	0.5935						
J Test[29]	0.0000		0.0000		0.6582		0.0000		0.5428	
JBT[30] Residuals	13.5574	0.0011	14.5508	0.0007	2.4642	0.2917	0.0369	0.9817	0.5136	0.7735

APPENDIX 7. GENERALIZED METHOD OF MOMENTS RESULTS - MULTIPLE REGRESSION - MONTHLY VARIABLE LEVELS

[28] The Sargan-Hansen test (or J test) is a statistical analysis employed to test over-identification constraints in a statistical model, which can be used in the Generalized Method of Moments with time series, as is the context of significance of the Skew Index and other fear indexes with the S&P500. The Null Hypothesis of J Test is that the Model is valid identifying restrictions, otherwise the alternative hypothesis is accepted, where the data does not come close to finding said restrictions. (Bhargava, 1991; L. P. Hansen, 1982; Sargan, 1958)

[29] The J Test null hypothesis is that the model is valid identifying restrictions. Alternative hypothesis: Data does not come close to finding the restrictions.

[30] Jarque Bera Test: Normality test. Ho: the data series has kurtosis = 3 and asymmetry = 0

Panel A: Generalized Method of Moments - Simple | Dependent Variable: S&P500 Monthly Log Returns				
	Model 1.1		Model 2.1	
	Value	P-Value	Value	P-Value
Intercept	0.0058	0.0143	0.0058	0.0088
Skew Index	-0.0647	0.1013		
VIX			-0.0162	0.3155
J[31] Test	0.0000		0.0000	
Jarque Bera Test Residuals	82.4665	0.0000	64.9662	0.0000

Appendix 8. Generalized Method of Moments results - simple regression – monthly variable log returns

Panel B: Generalized Method of Moments - Multiple | Dependent Variable: S&P500 Monthly Log Returns								
	Model 3.1		Model 5.1		Model 6.1		Model 7.1	
	Value	P-Value	Value	P-Value	Value	P-Value	Value	P-Value
Intercept	0.0058	0.0102	0.0050	0.0696	0.0049	0.0736	0.0051	0.0605
Skew Index	-0.0703	0.0862	-0.0714	0.1496	-0.0709	0.1574	-0.0721	0.1524
VIX	-0.0176	0.2779					-0.0124	0.5087
IVX			0.0741	0.2897	0.0829	0.2299	0.0509	0.5004
XAU					-0.0151	0.5865	-0.0163	0.5570
Bull Bear Spread								
J[32] Test	0.0000		0.0000		0.0000		0.0000	
JBT[33] Residuals	61.9603	0.0000	54.2293	0.0000	57.1428	0.0000	52.2762	0.0000

Appendix 9. Generalized Method of Moments results - Multiple regression - monthly variable log returns

[31] The J Test null hypothesis is that the Model is valid when identifying the restrictions, otherwise the alternative hypothesis is accepted, where the data does not come close to finding the restrictions. (Bhargava, 1991; L. P. Hansen, 1982; Sargan, 1958)

[32] The J Test null hypothesis is that the Model is valid when identifying the restrictions. (Bhargava, 1991; L. P. Hansen, 1982; Sargan, 1958)

[33] Jarque Bera Test: Normality test. Ho: the data series has kurtosis = 3 and asymmetry = 0.

	MA(4) | sGARCH(1,3)			MA(4) | eGARCH(1,3)			MA(4) | gjrGARCH(1,3)			MA(4) | iGARCH(1,3)		
Innovations Dist	Normal	t Student	sstd	Normal	t Student	sstd	Normal	t Student	sstd	Normal	t Student	sstd
						Information Criteria						
Akaike	-3.633	-3.6382	-3.6325	-3.6696	-3.6651	-3.6593	-3.6327	-3.6392	-3.6335	-0.78609	-0.7705	-0.76412
Bayes	-3.5332	-3.5273	-3.5104	-3.5587	-3.5431	-3.5262	-3.5218	-3.5172	-3.5004	-0.69734	-0.6707	-0.65319
Shibata	-3.6343	-3.6398	-3.6344	-3.6712	-3.667	-3.6616	-3.6343	-3.6411	-3.6358	-0.78712	-0.7718	-0.76572
Hannan-Quinn	-3.5933	-3.5941	-3.5839	-3.6255	-3.6165	-3.6063	-3.5886	-3.5906	-3.5805	-0.75075	-0.7308	-0.71996
						Ljung-Box Test on Standardized Residuals P-values						
Lag[1]	0.6811	0.5960	0.5961	0.4928	0.4249	0.4274	0.6085	0.5139	0.5099	2.07E-04	4.70E-05	4.66E-05
Lag11]	1	1	1	1	1	1	1	1	1	0.00E+00	0.00E+00	0.00E+00
Lag [19]	0.7773	0.7693	0.7693	0.7811	0.7618	0.7626	0.7242	0.7104	0.7081	2.20E-07	1.96E-08	1.93E-08
						Nyblom Stability Test Statistics [34]						
Joint Statistics	4.0676	3.7658	3.8247	2.3106	2.5678	2.6285	4.1153	3.4526	3.4751	129.5047	129.0463	129.1572
ma1	0.1263	0.0780	0.0779	0.0398	0.0290	0.0285	0.1250	0.0764	0.0782	0.4104	0.1683	0.16787
ma2	0.2270	0.1782	0.1782	0.0830	0.0707	0.0701	0.2168	0.1617	0.1614	0.0052	0.0480	0.0479
ma3	0.1164	0.1300	0.1300	0.0315	0.0328	0.0325	0.1149	0.1289	0.1297	0.0164	0.0476	0.047
ma4	0.2392	0.3169	0.3169	0.0849	0.0825	0.0824	0.2401	0.3031	0.3039	0.2260	0.2110	0.21426
Omega	0.2798	0.3462	0.3463	0.5476	0.5578	0.5587	0.1995	0.2392	0.2377	112.5000	112.5230	112.52221
alpha1	0.5726	0.6963	0.6964	0.0645	0.0619	0.0613	0.3905	0.4724	0.4721	100.5000	100.9587	100.95995
beta1	0.6569	0.7226	0.7226	0.4615	0.4713	0.4722	0.5358	0.5916	0.5879	42.9500	41.2296	41.21906
beta2	0.4920	0.5538	0.5538	0.5154	0.5236	0.5242	0.3564	0.4068	0.4064	40.1200	38.3026	38.2913
beta3	0.5690	0.6657	0.6658	0.5050	0.5144	0.5155	0.4348	0.5298	0.5283			
gamma1				0.3402	0.2774	0.2789	0.2009	0.2264	0.2272			
Skew			0.1297			0.0924			0.1250			0.09484
Shape		0.1995	0.1994		0.1182	0.1166		0.1640	0.16433		107.0087	107.01446

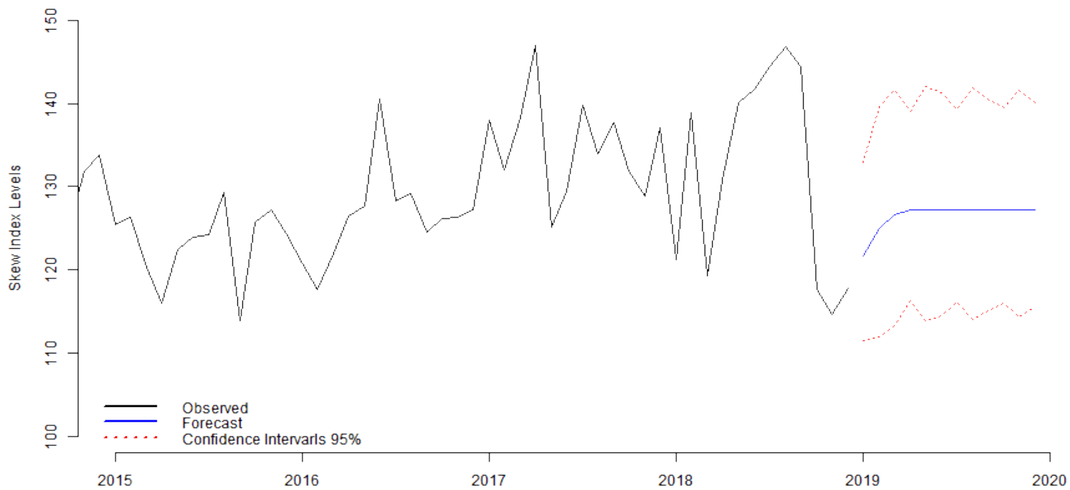
[34] For the Nyblom stability test, the joint statistics' Asymptotic Critical Values at 95% confidence level is 2.75, while for each of the individual statistics it is 0.47.

(Continuation Appendix 10)

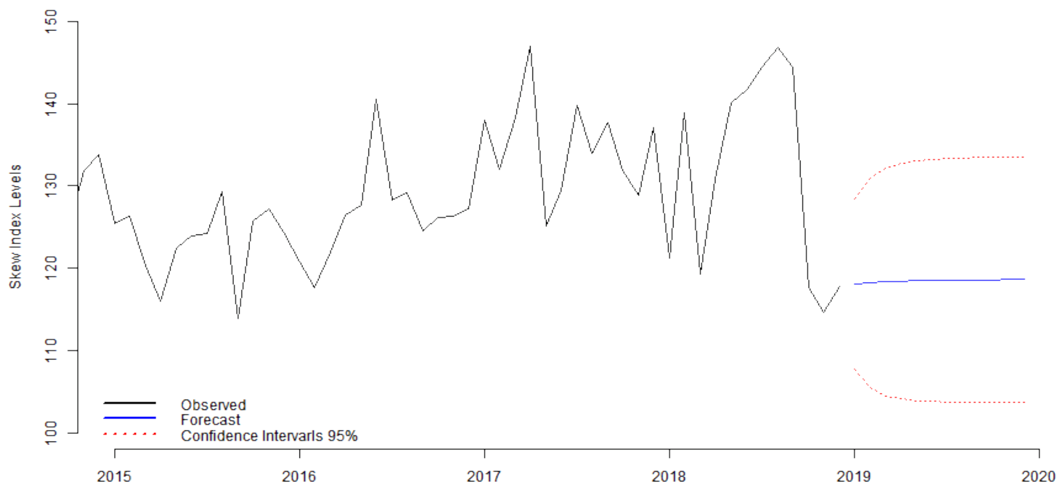
	Engle-Ng test for asymmetric effects T-values											
Sign Bias	0.9164	1.0549	1.0549	0.5336	1.3514	1.3522	0.4880	1.3209	1.3241	0.5047	0.4561	0.456
Neg Sign Bias	0.4904	0.546	0.546	0.7548	1.2052	1.2042	0.5894	1.1008	1.1063	2.9856	2.8842	2.8838
Pos Sign Bias	0.1133	0.1587	0.1587	0.1536	0.6005	0.6007	0.2375	0.7509	0.7531	0.2144	0.1814	0.1813
Joint Effect	1.0941	1.4322	1.4322	0.6066	2.209	2.209	0.4129	2.0551	2.0693	10.7585	10.0982	10.0962
	Engle's Lagrange Multiplier P-Value											
LM Engle[35]	0.05560	0.05182	0.05183	0.59720	0.05112	0.05120	0.04979	0.04776	0.04754	0.05563	0.05236	0.05238
	Time Series' Cross Validation											
Success Rate	92.4856%	92.4856%	92.4856%	93.0636%	93.0636%	93.0636%	92.4856%	92.4856%	92.4856%	100%	100%	100%
MAE	3.40E-02	3.40E-02	3.40E-02	3.39E-02	3.40E-02	3.40E-02	3.39E-02	3.40E-02	3.40E-02	3.40E-02	3.40E-02	3.40E-02
MSE	2.40E-06	2.46E-06	2.46E-06	2.43E-06	2.45E-06	2.45E-06	2.39E-06	2.41E-06	2.41E-06	2.40E-06	2.44E-06	2.44E-06
RMSE	1.55E-03	1.57E-03	1.57E-03	1.56E-03	1.56E-03	1.56E-03	1.54E-03	1.55E-03	1.55E-03	1.55E-03	1.56E-03	1.56E-03

APPENDIX 10. COMPARATIVE RESULTS GARCH MODELS AND INNOVATIONS' DISTRIBUTIONS

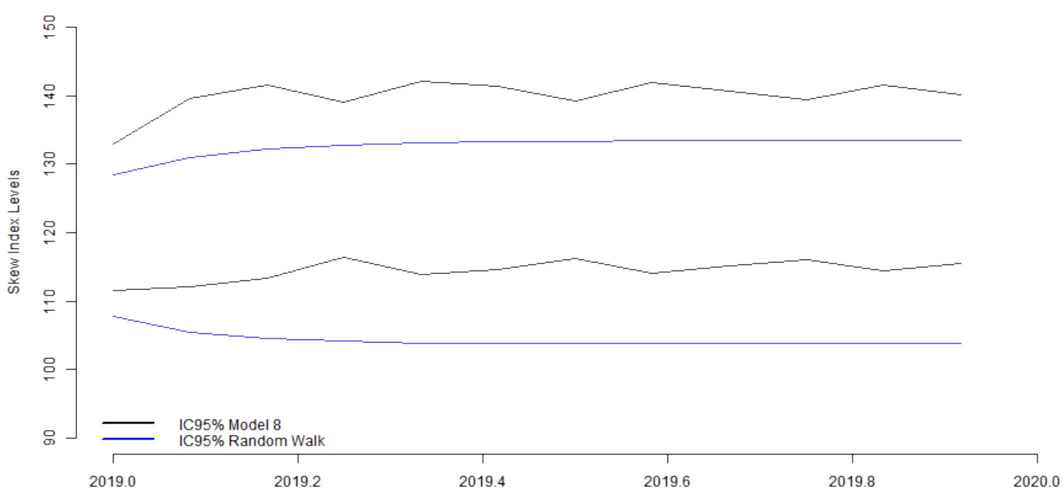
[35] The null hypothesis of Engle's Lagrange Multiplier test is the NO existence of GARCH effects; for the particular case of evaluations of GARCH models, rejecting the null hypothesis would indicate that said model does not adjust adequately to the time series.



APPENDIX 11. 12 MONTHS AHEAD FORECAST – MODEL 8. SOURCE: THE AUTHOR.



APPENDIX 12. 12 MONTHS AHEAD FORECAST - RANDOM WALK. SOURCE: THE AUTHOR.



APPENDIX 13. FORECAST DENSITY COMPARISON 95%. SOURCE: THE AUTHOR.