

## The out-sample predictive power of convenience yields

### Abstract

This article explores the predictive power of convenience yields with respect to future spot prices of five industrial metals: aluminum, copper, lead, and zinc. To that end, alternative measures of convenience yields are considered. The estimation results show that convenience yields may have predictive power but within short-time horizons of 1, 3, and 6 months. Furthermore, the methodology utilized to compute the convenience yield may play a role in such findings. For longer prediction horizons (e.g., 12 months), however, interest rates and nominal exchange rates may be more relevant to explaining the evolution of future spot prices of mineral commodities.

JEL classification: Q02, L72, C13

Keywords: theory of storage; interest-adjusted basis; convenience yield.

### 1 Introduction

In their influential article, Fama and French (1987) assessed the forecast performance of commodity futures prices on the basis of two models: the theory of storage and expected premium. The former explains the difference between the futures and spot prices (basis) in terms of interest rates, warehouse costs, and convenience yields. The expected premium model in turn splits a futures price into an expected premium and the current expectation of the change in the spot price. Fama and French found more evidence in favor of futures prices responding to storage-cost factors than in favor of futures prices containing premiums or having the power to forecast spot prices.

More recent contributions on the relationship between commodity futures and spot prices are the survey article by Chow, McAleer, and Sequeira (2000), and various empirical applications by Chernenko, Schwarz, and Wright (2004), Movassagh and Modjtahedi (2005), Modjtahedi and Movassagh (2005), Watkins and McAleer (2006), Alquist and Killian (2010), Reeve and Vigfusson (2011), Chinn and Coibion (2014), Xiao, Colwell, and Bhar (2015), Fernandez(2017) among others.

Another strand of the literature has focused on the empirical relationship between changes in commodity prices and inflation (e.g., Marquis and Cunningham 1990; Furlong and Ingenito 1996; Celasun, Ratnovski, and Mihet 2012). More recently, Gospodinov and Ng (2013) found that the two leading principal components of 23 commodity convenience yields<sup>1</sup> derived from the cost-of-carry model had predictive power for inflation in the G7 countries, after controlling for the unemployment gap (i.e., unemployment rate minus its long-term level) and the oil price, at a 1-, 3-, 6-, and 12-month horizon. The authors argue that convenience yields can be seen as informational variables about future economic conditions.

This article is somehow connected to Gospodinov and Ng (2013), but it focuses on the forecastability of spot prices of five industrial metals (aluminum, copper, lead, and zinc) by considering three alternative measures of convenience yields: the

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<sup>1</sup> The benefit or premium associated with holding an underlying product or physical good, as opposed to the associated derivative contract (source: <https://www.investopedia.com/terms/c/convenienceyield.asp>).

one implied by the cost-of-carry model (e.g., Fama and French 1988; Gospodinov and Ng op cit.), Heaney (2002), and West (2012). The latter, as opposed to the two former, ensures a non-negative convenience yield.

This article is organized as follows. Section 2 briefly discusses the three methods utilized to compute convenience yields, and it presents the econometric specification used to forecast spot prices. Section 3 presents the data while Section 4 presents the empirical results. Section 5 closes by presenting the main findings.

## 2 Methodological aspects

### 2.1 Measurement of convenience yields

The well-known cost-of-carry model establishes an arbitrage relation between futures and spot prices. Specifically, the cost of carry measures the storage cost plus the interest that is paid to finance the asset less the income earned on the asset. The latter can take the form of a so-called convenience yield: the benefit associated with holding the physical commodity (see, for instance, Hull 2014, chapter 5, section 5.12). In the absence of arbitrage, the relation between the futures price at  $t$  for delivery at  $T$ ,  $F_{t,T}$ , and the spot price at  $t$ ,  $S_t$ , is given by

$$F_{t,T} = S_t \exp[(r_t + u_t - y_t)\tau] \quad (1)$$

where  $(r_t + u_t - y_t)$  represents the cost of carry, in that  $r_t$ ,  $u_t$ , and  $y_t$  are, respectively, the risk-free rate, the storage cost rate, and the convenience yield at time  $t$ , and  $\tau \equiv (T-t)$  is the time remaining until maturity.

Intuitively, the convenience yield reflects the market's expectations about the future availability of the commodity. The greater the likelihood of a future shortage, the higher the convenience yield.

An approximation to the convenience yield ( $y_t$ ), which was derived by Heaney (2002), is given by<sup>2</sup>

$$y_t = \ln \left\{ \left( 2 + \frac{\sigma_S^2 \tau}{2} \right) \Phi \left( \frac{\sqrt{\sigma_S^2 \tau}}{2} \right) + \frac{\sqrt{\sigma_S^2 \tau}}{2\pi} \exp \left( -\frac{\sqrt{\sigma_S^2 \tau}}{8} \right) \right\} \\ - \ln \left\{ \left( 2 + \frac{\sigma_F^2 \tau}{2} \right) \Phi \left( \frac{\sqrt{\sigma_F^2 \tau}}{2} \right) + \frac{\sqrt{\sigma_F^2 \tau}}{2\pi} \exp \left( -\frac{\sqrt{\sigma_F^2 \tau}}{8} \right) \right\} \quad (2)$$

where  $\sigma_S$  and  $\sigma_F$  are the volatilities of the spot and futures returns, respectively, and  $\Phi(\cdot)$  is the cumulative distribution function of a standard normal.

One disadvantage of Heaney's formula is that it does not preclude the convenience yield from taking on negative values. In order to tackle this issue, West (2012) derived a formula to compute convenience yields based on option pricing.

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<sup>2</sup> Later research by Dockner, Eksi and Rammerstorfer (2015), for commodities that exhibit mean reversion, discusses the obtainment of convenience yields by means of arithmetic- and geometric-mean Asian options.

Applications of this approach are in Omura and West (2014), and in Omura et al. (2015). Specifically, West's formula is given by

$$y_t = S_t^* \Phi(d_1) - F_{t,T} \Phi(d_2) \quad (3)$$

where  $S_t^* = S_t + U_{t,T}$ ,  $U_{t,T}$  is the total storage cost between  $t$  and  $T$ ,  $d_1 = \frac{\ln(F_T) + 0.5\tau\sigma_c^2}{\sigma_c\sqrt{\tau}}$ ,  $d_2 = d_1 - \sigma_c\sqrt{\tau}$ ,  $F_T = S_t^*/F_{t,T}$ ,  $\sigma_c^2 = \sigma_{S^*}^2 - 2\sigma_{S^*}\sigma_{F_{t,T}}\rho_{S^*F_{t,T}} + \sigma_{F_{t,T}}^2$ ,  $\sigma_{S^*}^2$  and  $\sigma_{F_{t,T}}^2$  are the variances of  $S^*$  and the futures price, respectively, and  $\rho_{S^*F_{t,T}}$  is the correlation coefficient between the two latter.

Figure 1 illustrates the use of these formulas for 3-month copper futures during the period of February 1983–December 2017 (monthly average based on daily figures). For the sake of comparison, the convenience yield computed from the cost-of-carry model is also depicted. The latter can be approximated under discrete compounding as (e.g., Fama and French 1988; Gospodinov and Ng 2013)

$$y_t = \frac{(1+r_{t,T})S_t + U_{t,T} - F_{t,T}}{S_t} \quad (4)$$

where  $r_{t,T}$  are  $U_{t,T}$  the risk-free rate and the marginal storage cost between  $t$  and  $T$ , respectively.<sup>3</sup>

As can be seen from the graph, along the sample period Heaney and the cost-of-carry formulae yielded some negative values. For instance, the minimum value for the former was  $-3.0\%$  while  $-0.29\%$  for the latter. (West's minimum was  $0.10\%$ ). Moreover, Heaney estimate tended to be more conservative than those of the two other methods. Indeed, during the period of February 1983-December 2017, the median convenience yield values for the cost-of-carry, West, and Heaney were  $1.83\%$ ,  $1.20\%$ , and  $0.34\%$ , respectively, while the mean values,  $2.77\%$ ,  $2.36\%$ , and  $0.84\%$ , respectively. In addition, during the sample period Heaney estimate was considerably less volatile than the those of the cost-of-carry and West, as measured by the inter-quartile range:  $0.95\%$  versus  $2.69\%$  and  $2.28\%$ , respectively (details are provided in Table 1).

From Equation (1), one can obtain the interest-adjusted basis at time  $t$  ( $iab_t$ ) under continuous compounding, that is, the difference between the log-basis,  $\ln(F_{t,T}/S_t)$ , and the interest rate over the  $\tau$ -period:

$$\ln(F_{t,T}/S_t) - r_t \tau = (u_t - y_t)\tau \equiv iab_t \quad (5)$$

In the absence of inventory costs,  $iab$  is proportional to the negative convenience yield.

Under the theory of storage, Fama and French (1988) assume that the (relative) convenience yield<sup>4</sup> is a convex function of the inventory level, whereas  $iab_t$  is a concave function of the latter. In particular, at low inventory levels, the convenience

<sup>3</sup> One reason for using this approximation is that warehousing charges for industrial metals are provided on a ton-per-day basis by the London Metal Exchange.

<sup>4</sup> The relative convenience yield is defined as the ratio of the convenience yield to the spot price. In particular, Fama and French consider equation (5) under discrete compounding,  $(F_{t,T}-S_t)/S_t - r_t\tau = (U_{t,T}-Y_t)/S_t$ , where  $U_{t,T}$  and  $Y_t$  are the total inventory cost and convenience yield, respectively.

yield exceeds the storage cost rate (i.e.,  $y_t > u_t$ ) and  $iab_t < 0$ ; at high inventory levels, the convenience yield falls toward zero, and  $iab_t$  increases toward the storage cost rate at a decreasing rate.

The association between copper  $iab_t$  for 3-month futures and inventory (measured in days of world consumption) is illustrated in Figure 2 for the period of April 1986–December 2017. As can be seen, the figure fits very accurately the above description. Figure 3 in turn depicts annual series of copper convenience yield for 3-month futures and inventory for the period of 1983–2017. It is apparent the negative association between the convenience yield and inventory.

## 2.2 Predictive regression of spot prices

A complement to the theory to the storage for explaining the relationship between spot and futures prices is the theory of normal backwardation (e.g., Kolb and Overdahl, 2007, chapter 5). This states that risk-averse investors earn a risk premium for the fluctuations in future spot prices. Based on this theory and the work by Chen, Rogoff, and Rossi (2010), which found that exchange rates forecast commodity prices, Gospodinov and Ng (2013) proposed the following model for the expected future return on the spot:

$$E_t \Delta^\tau s_{t+\tau} = \beta_1 y_{t,\tau} + \beta_2 r_{t,\tau} + \beta_3 \Delta^\tau x_t^k + \varepsilon_{t,\tau} \quad (6)$$

where  $\Delta^\tau s_{t+\tau} = (E_t S_{t+\tau} - S_t) / S_t$ ,  $S_t$  is the spot price at time  $t$ ,  $y_{t,\tau}$  and  $r_{t,\tau}$  are the convenience yield and the nominal risk-free rate between  $t$  and  $(t + \tau) = T$  (i.e., futures contract delivery date), and  $x_t^k$  is the nominal exchange rate between the United States and country  $k$ .

## 3 Data

Spot and 3-month futures prices of aluminum, copper, lead, nickel, and zinc in US dollars are from the London Metal Exchange (LME), via Bloomberg, and from Christopher Gilbert's website, <https://sites.google.com/site/christopherlesliegilbert/home>. The sample period under consideration is January 1983-December 2017, which has complete information for the five metals. Tin, another industrial metal traded at LME, is excluded from the sample because of missing information on spot and 3-month futures prices from November 1985 to August 1989.

Historical information for interest and exchanges rates is from FRED, <http://research.stlouisfed.org/fred2/>. In particular, a proxy for the risk-free rate is the 3-month T-Bill rate and the exchange rate is the real trade weighted U.S. Dollar index: major currencies.

## 4 Empirical results

### 4.1 Out-of-sample predictive power of convenience yields

The empirical specification is based on the predictive regressions for individual commodities considered by Gospodinov and Ng op. cit (Table 7 of their article)<sup>5</sup>:

$$\Delta s_{t+h} = \beta_0 + \beta_1 \Delta s_t + \beta_2 \Delta s_{t-1} + \beta_3 y_t + \beta_4 r_t + \beta_5 \Delta x_t + \varepsilon_t \quad (7)$$

where  $\Delta s_{t+h} = \ln(S_{t+h}/S_t)$  is the log-return on the spot price between  $t$  and  $(t+h) \approx (S_{t+h} - S_t)/S_t$ ,  $\Delta s_t$  is the log-return on the spot price between  $(t-1)$  and  $t$ ,  $\Delta s_{t-1}$  is its lagged value,  $y_t$  is the convenience yield computed at time  $t$  from 3-month futures contracts,  $r_t$  is the 3-month T-Bill rate observed at time  $t$ , and  $\Delta x_t$  is the log-return on the real trade weighted U.S. Dollar index: major currencies between  $(t-1)$  and  $t$ .

Predictive equations for  $h = 1, 3, 6,$  and  $12$  months ahead are shown in Panels (a) through (d) of Table 2 for aluminum, copper, lead, nickel, and zinc. Each panel in turn depicts the estimation results for one specific method used to compute the convenience yield: (i) cost-of-carry, (ii) Heaney, and (iii) West.  $R^2$  are reported at the bottom of each regression model. As can be seen,  $R^2$  ranges from 1% to 17%, depending on the prediction horizon and the metal in question. Largest  $R^2$  are observed at  $h = 1$  for copper—17% under Heaney and West convenience yield, and lead—13% under cost-of-carry convenience yield.

The out-of-sample predictive power occurs mostly at  $h=1$  and  $3$ , and, to a lesser extent at  $h = 6$ . At  $h=1$  (Panel (a)), for instance, most statistically significant cases of ( $p$ -value  $\leq 0.1$ ) take place under Heaney (subpanel (ii)—aluminum, copper, and nickel, followed by West (subpanel (iii))—copper and zinc, and cost-of-carry (subpanel (i))—zinc. Just to have a flavor of the magnitudes, in subpanel (ii), an increment of 1-percent of the convenience yield today would lead to a decrement of 0.41 percent points in the aluminum spot return 1 month into the future.

It is worth pointing out that Gospodinov and Ng op. cit fitted one-month predictive regressions to five metals for the period of March 1983–July 2008: copper, gold, palladium, platinum, and silver (Table 7 in their article). Only for palladium was the convenience yield found to be statistically significant ( $\hat{\beta}_3 = -0.36$ ).

At  $h = 3$  (Panel (b)), there are only 4 cases in which the convenience has out-of-sample predictive power: zinc (cost-of-carry and West) and copper and lead (Heaney). However, its marginal impact on the return is greater than at  $h = 1$ . For instance, in the case of zinc (cost-of-carry), an increment of 1 percent point on the convenience yield today would lead to a decrease of 1.2 percent points of the spot return in a 3-month forecast horizon. At  $h = 6$  (Panel (c)), the number of significant

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<sup>5</sup> This specification is analyzed only for  $h = 1$  by the authors.

cases drops to three: zinc (cost-of-carry and West) and nickel (Heaney). The negative impact on the convenience yield is still numerically large with respect to the out-of-sample spot log-return (e.g.,  $\hat{\beta}_3 = -1.97$  for zinc under West methodology).

The intuition for an inverse association between expected commodity price changes and convenience yields is provided by Fama and French (1988). Specifically, a permanent demand shock has a large impact on spot prices when inventory levels are low, but a smaller effect on expected spot prices because the market anticipates future demand and supply responses. Therefore, higher convenience yields and lower inventory levels will be generally associated with lower expected spot prices.

At  $h = 12$  (Panel (d)), however, convenience yields do not exhibit any out-of-sample predictive power, except for nickel at the 10% significance level. By contrast, in this case the cost-of-carry convenience yield has a positive impact on the predicted 12-month return: 0.97-percent point increase per 1-percent increase in the convenience yield. For aluminum, copper, and zinc, the 3-month interest rate and/or the one-month log-return on the real trade weighted U.S. Dollar index may be more relevant drivers than convenience yields. This is particularly so for aluminum under the three convenience yield specifications, where both variables have a negative impact on the predicted 12-month return. For instance, a 1-percent point increase in the 3-month interest rate today would translate into a decrease of 2.1-percent points of the aluminum spot log-return one year from today. The negative impact of 1-percent point increase in the exchange rate proxy is of a similar magnitude ( $\approx -2.0$ ).

In the case of lead, under the three convenience yield specifications, the only relevant explanatory variable is the spot log-return between  $(t-1)$  and  $t$ . That is,  $\widehat{\Delta s_{t+12}} = \hat{\beta}_1 \Delta s_t$ , where  $\hat{\beta}_1$  ranges from 0.57 to 0.61. On the other, under the West specification, nor the constant term and any regressor are statistically significant in the case of nickel. This means that,  $\widehat{\Delta s_{t+12}} = 0$  or  $\widehat{s_{t+12}} = s_t$ , that is, this is the no-change model or random-walk hypothesis.<sup>6</sup>

#### 4.2 Root-mean squared forecast error (RMSFE)

Table 3 presents RMSFE for regression model (7) relative to the no-change or random-walk model for the full sample. As can be seen the minimum relative RMSFE for copper, lead, and zinc is reached at  $h = 1$  under all of the convenience yield specifications. In particular, the minimum RMSFE at  $h = 1$  for lead is provided by the cost of carry, while the maximum by West. In the case of zinc, it occurs exactly the opposite at  $h = 1$ . Moreover, West methodology yields consistently smaller relative RMSFE at all prediction horizons for zinc. By contrast, no significant differences across methods are observed for copper at  $h = 1$ . In the case of aluminum, the minimum relative RMSFE is reached at  $h = 6$  under

<sup>6</sup> From equation (7), this would imply that  $s_{t+1} = s_t + \varepsilon_t$ , where  $s_t \equiv \ln(S_t)$  is the log-spot price. By subsequent substitutions, one obtains that  $s_{t+h} = s_t + \sum_{i=1}^h \varepsilon_{t+i}$ . Hence  $E(s_{t+h}|s_t) = s_t$ .

the three methods. Nevertheless, the evidence is less clear-cut for nickel. For instance, the minimum relative RMSFE is reached at  $h = 3$  under Heaney, but at  $h = 1$  under the cost-of-carry and West.

All in all, these results are in line with the evidence of the previous section in that most regressors of equation (7) tend to be more strongly associated with  $\Delta s_{t+h}$  at  $h = 1, 3,$  and  $6$ . However, there are still some forecasting gains of using such a regression model relative to a no-change model at  $h = 12$ , as the relative RMSFE is less than 1.

Table 4 presents recursive relative RMSFEs: the first 15 years of data are used to estimate regression model (7), and successively one observation is added and the model re-estimated. That is, the pseudo out-of-sample evaluation period covers January 1998-December 2017. In general, the results show that most forecasting gains of re-estimating the model through time, relative to Table 3, arise at  $h = 12$ . For instance, under the cost-of-carry, the relative RMSFE of Table 4 at  $h = 12$  are 0.922, 0.940, 0.951, 0.967, and 0.959 for aluminum, copper, lead, nickel, and zinc, respectively, as opposed to 0.966, 0.970, 0.984, 0.979, and 0.970 of Table 3. This finding suggests that the regression model at longer forecasting horizons may have experienced some structural changes along the pseudo out-of-sample evaluation period.

## 5 Conclusions

This article studied the predictive power of convenience yields with respect to future spot prices of five industrial metals—aluminum, copper, lead, and zinc— for the period of January 1983-December 2017. To that end, alternative measures of convenience yields were considered: cost-of-carry, Heaney (2002), and West (2012). The estimation results show that convenience yields may have predictive power but within short-time horizons of 1, 3, and 6 months. Furthermore, the methodology utilized to compute the convenience yield may play a role in such findings. For longer time horizons (e.g., 12 months), however, interest rates and nominal exchange rates may be more relevant to explaining the evolution of future spot prices.

In addition, root-mean squared forecast error (RMSFE) calculations for the full sample show that most forecasting gains of using convenience yields, interest rates, and exchanges rates as predictors, relative to a no-change model are reached at forecasting horizons of 1, 3, and 6 months. On the hand, recursive relative RMSFE show that re-estimating the regression model of equation (7) provides more forecasting gains at longer time horizons.

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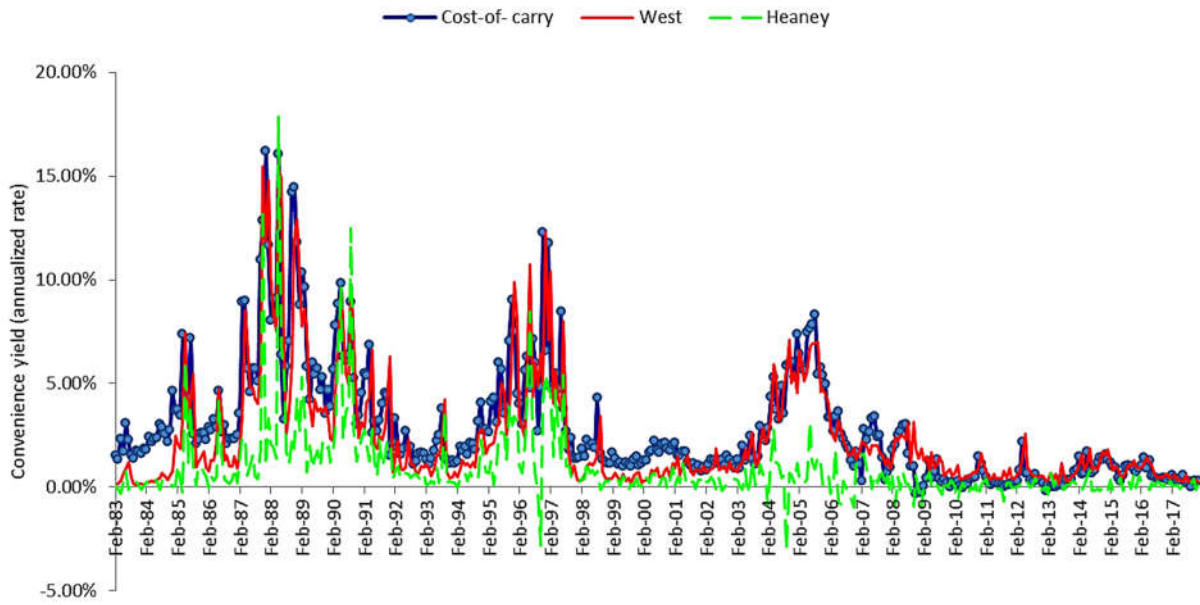
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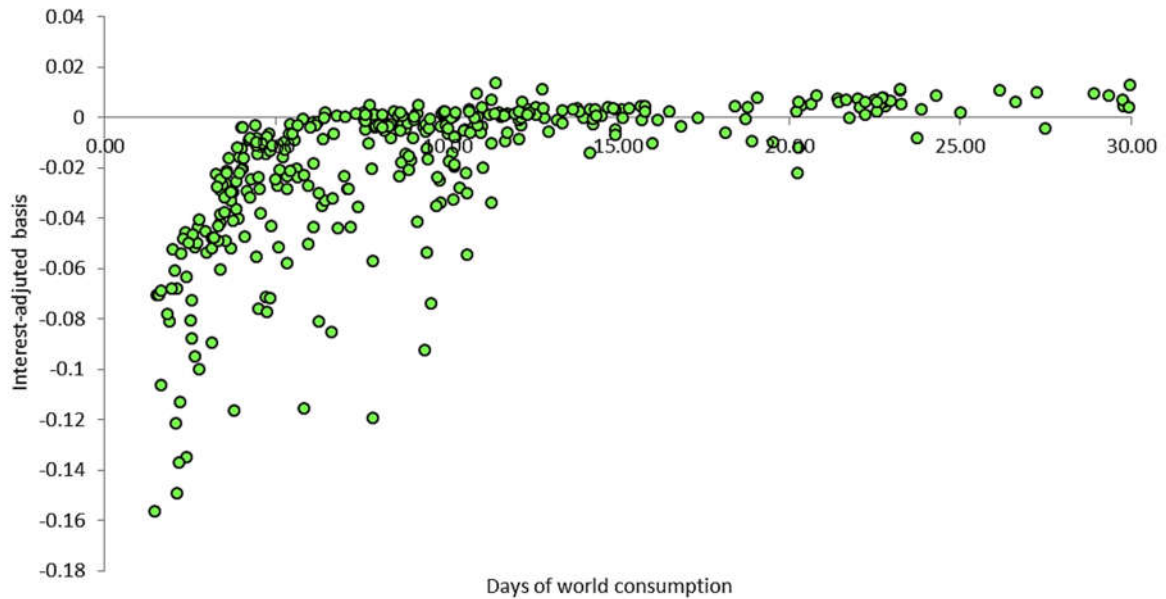
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**Figure 1** Monthly series of copper convenience yield: February 1983–December 2017



Note: Own elaboration on the basis of information from Bloomberg and the London Metal Exchange.

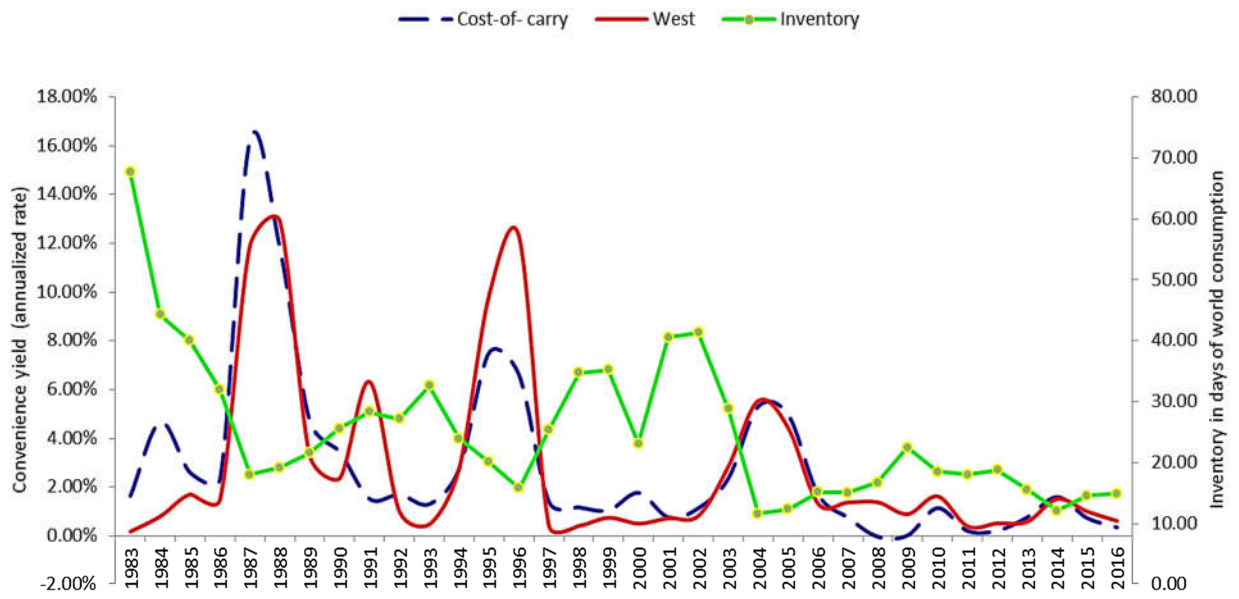
**Figure 2** Interest-adjusted basis and stock level of copper: April 1986–December 2017



Note: Own elaboration on the basis of information from Bloomberg, FRED and various issues of the World Bureau of Metal Statistics Yearbook.

**Figure 3**

Convenience yield and inventory series of copper: 1983–2017



Note: Own elaboration on the basis of information from Bloomberg and various issues of the World Bureau of Metal Statistics Yearbook. Annual convenience yield series are averages of monthly figures.

**Table 1** Descriptive statistics of convenience yields (annualized rates): February 1983–December 2017

	Al			Cu			Pb		
	CC	West	Heaney	CC	West	Heaney	CC	West	Heaney
Mean	2.54%	2.07%	0.81%	2.77%	2.36%	0.87%	1.54%	1.90%	0.73%
Median	2.06%	1.36%	0.35%	1.83%	1.20%	0.34%	0.34%	0.85%	0.30%
Min	-0.68%	0.08%	-0.87%	-0.29%	0.10%	-3.01%	-1.62%	0.00%	-1.07%
Max	29.09%	28.59%	15.10%	16.21%	15.45%	17.84%	26.65%	27.36%	14.43%
IQ range	1.41%	1.30%	0.57%	2.69%	2.28%	0.95%	1.85%	1.62%	0.85%

	Ni			Zn		
	CC	West	Heaney	CC	West	Heaney
Mean	6.07%	7.09%	1.54%	3.03%	2.67%	0.95%
Median	6.24%	6.17%	0.83%	2.15%	1.76%	0.49%
Min	-3.93%	0.97%	-11.24%	-1.55%	0.16%	-22.05%
Max	46.00%	45.49%	64.52%	16.90%	16.27%	22.57%
IQ range	5.44%	3.53%	1.48%	2.09%	1.94%	0.80%

Note: Estimates are based on equations (2)-(4) in the text. IQ range is the inter-quartile range =  $Q_3 - Q_1$ .

**Table 2** Forecast of future spot price changes: February 1983–December 2017

(a)  $h = 1$ -month ahead

Dependent variable:  $\Delta S_{t+h}$

(i) Cost-of-carry convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.005	1.071	0.284	0.004	0.847	0.397	0.002	0.335	0.737	0.005	0.847	0.397	0.006	1.210	0.226
$\Delta S_t$	0.158	2.056	0.040	0.425	6.154	0.000	0.312	5.928	0.000	0.170	2.452	0.014	0.298	5.964	0.000
$\Delta S_{t-1}$	0.071	1.047	0.295	-0.094	-1.782	0.075	-0.075	-1.245	0.213	-0.031	-0.500	0.617	0.011	0.204	0.838
$y_t$	-0.241	-1.042	0.297	-0.020	-0.180	0.857	0.413	1.460	0.144	-0.029	-0.166	0.868	-0.303	-1.979	0.048
$r_t$	0.040	0.325	0.745	-0.038	-0.358	0.721	-0.190	-1.271	0.204	-0.006	-0.028	0.977	0.154	1.176	0.239
$\Delta x_t$	-0.330	-1.811	0.070	-0.083	-0.517	0.605	-0.267	-1.064	0.287	-0.431	-1.674	0.094	-0.040	-0.224	0.823
	$R^2 = 0.05$			$R^2 = 0.16$			$R^2 = 0.13$			$R^2 = 0.05$			$R^2 = 0.09$		

(ii) Heaney convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.004	0.873	0.383	0.004	0.837	0.403	0.002	0.295	0.768	0.006	1.149	0.251	0.004	0.796	0.426
$\Delta S_t$	0.120	1.340	0.180	0.429	6.355	0.000	0.319	6.368	0.000	0.174	2.814	0.005	0.291	6.128	0.000
$\Delta S_{t-1}$	0.074	1.089	0.276	-0.090	-1.703	0.089	-0.063	-1.050	0.294	-0.017	-0.272	0.785	0.017	0.306	0.759
$y_t$	-0.409	-1.661	0.097	-0.312	-2.185	0.029	0.539	1.028	0.304	-0.289	-5.415	0.000	-0.466	-1.368	0.171
$r_t$	0.003	0.027	0.978	0.020	0.206	0.837	-0.117	-0.802	0.423	0.046	0.426	0.670	0.083	0.702	0.483
$\Delta x_t$	-0.341	-1.870	0.061	-0.077	-0.482	0.630	-0.278	-1.098	0.272	-0.383	-1.566	0.117	-0.011	-0.064	0.949
	$R^2 = 0.05$			$R^2 = 0.17$			$R^2 = 0.12$			$R^2 = 0.07$			$R^2 = 0.11$		

(iii) West convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.005	0.903	0.366	0.006	1.199	0.230	0.001	0.187	0.852	0.017	1.975	0.048	0.012	2.471	0.013
$\Delta S_t$	0.125	1.376	0.169	0.423	6.387	0.000	0.330	6.909	0.000	0.172	2.659	0.008	0.271	5.797	0.000
$\Delta S_{t-1}$	0.065	0.933	0.351	-0.081	-1.574	0.116	-0.051	-0.949	0.343	-0.022	-0.358	0.720	0.033	0.596	0.551
$y_t$	-0.105	-0.645	0.519	-0.162	-1.646	0.100	0.079	0.453	0.651	-0.171	-1.438	0.151	-0.685	-5.111	0.000
$r_t$	-0.046	-0.471	0.637	0.003	0.033	0.974	-0.034	-0.292	0.771	-0.032	-0.280	0.780	0.245	2.234	0.025
$\Delta x_t$	-0.346	-1.820	0.069	-0.078	-0.492	0.622	-0.274	-1.083	0.279	-0.403	-1.635	0.102	-0.101	-0.595	0.552
	$R^2 = 0.04$			$R^2 = 0.17$			$R^2 = 0.12$			$R^2 = 0.06$			$R^2 = 0.14$		

## (b) h = 3 months ahead

Dependent variable:  $\Delta S_{t+h}$ 

## (i) Cost-of-carry convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.015	1.024	0.306	0.021	1.186	0.236	0.008	0.423	0.673	0.027	1.410	0.159	0.025	1.512	0.130
$\Delta S_t$	0.265	2.117	0.034	0.491	3.402	0.001	0.355	3.398	0.001	0.161	1.344	0.179	0.422	3.875	0.000
$\Delta S_{t-1}$	0.123	1.192	0.233	-0.192	-1.728	0.084	-0.019	-0.146	0.884	-0.049	-0.502	0.616	0.133	1.018	0.309
$y_t$	-0.269	-0.531	0.595	-0.297	-0.910	0.363	0.470	0.889	0.374	-0.404	-1.004	0.316	-1.206	-2.430	0.015
$r_t$	-0.160	-0.507	0.612	-0.102	-0.280	0.779	-0.288	-0.711	0.477	0.245	0.465	0.642	0.553	1.405	0.160
$\Delta x_t$	-0.981	-2.264	0.024	-0.389	-0.758	0.449	-0.863	-1.485	0.138	-0.729	-1.309	0.191	-0.418	-0.884	0.377
	$R^2=0.06$			$R^2=0.05$			$R^2=0.05$			$R^2=0.03$			$R^2=0.08$		

## (ii) Heaney convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.014	1.019	0.308	0.019	1.054	0.292	0.008	0.409	0.682	0.020	1.183	0.237	0.017	1.049	0.294
$\Delta S_t$	0.217	1.412	0.158	0.483	3.461	0.001	0.374	3.675	0.000	0.156	1.434	0.152	0.374	3.523	0.000
$\Delta S_{t-1}$	0.140	1.352	0.176	-0.192	-1.690	0.091	0.008	0.065	0.948	-0.012	-0.121	0.903	0.122	0.919	0.358
$y_t$	-0.684	-1.143	0.253	-0.820	-2.392	0.017	0.195	0.224	0.823	-0.796	-5.378	0.000	-0.502	-0.898	0.369
$r_t$	-0.162	-0.571	0.568	-0.074	-0.221	0.825	-0.121	-0.299	0.765	0.106	0.311	0.756	-0.078	-0.241	0.810
$\Delta x_t$	-0.993	-2.307	0.021	-0.374	-0.718	0.473	-0.872	-1.483	0.138	-0.607	-1.161	0.245	-0.356	-0.745	0.457
	$R^2=0.07$			$R^2=0.06$			$R^2=0.05$			$R^2=0.07$			$R^2=0.05$		

## (iii) West convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.013	0.889	0.374	0.021	1.177	0.239	0.008	0.414	0.679	0.027	1.079	0.281	0.032	1.998	0.046
$\Delta S_t$	0.230	1.472	0.141	0.468	3.370	0.001	0.381	3.659	0.000	0.144	1.236	0.216	0.340	3.324	0.001
$\Delta S_{t-1}$	0.111	0.999	0.318	-0.187	-1.656	0.098	0.016	0.127	0.899	-0.043	-0.403	0.687	0.159	1.247	0.212
$y_t$	-0.071	-0.160	0.873	-0.211	-0.763	0.445	-0.025	-0.060	0.952	-0.154	-0.446	0.655	-1.298	-3.516	0.000
$r_t$	-0.265	-0.960	0.337	-0.187	-0.550	0.583	-0.075	-0.188	0.851	-0.108	-0.321	0.748	0.325	1.005	0.315
$\Delta x_t$	-0.997	-2.271	0.023	-0.381	-0.744	0.457	-0.871	-1.490	0.136	-0.713	-1.308	0.191	-0.514	-1.126	0.260
	$R^2=0.06$			$R^2=0.05$			$R^2=0.05$			$R^2=0.02$			$R^2=0.09$		

## (c) h = 6 months ahead

Dependent variable:  $\Delta s_{t+h}$ 

## (i) Cost-of-carry convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.027	0.985	0.325	0.042	1.266	0.206	0.020	0.528	0.598	0.031	0.851	0.395	0.052	1.651	0.099
$\Delta s_t$	0.204	1.269	0.205	0.454	2.794	0.005	0.389	2.251	0.024	0.241	1.328	0.184	0.562	2.735	0.006
$\Delta s_{t-1}$	0.075	0.506	0.613	-0.282	-1.535	0.125	0.055	0.291	0.771	0.107	0.636	0.525	0.198	1.047	0.295
$y_t$	0.231	0.255	0.799	0.065	0.102	0.919	-0.467	-0.681	0.496	0.148	0.323	0.746	-1.669	-2.192	0.028
$r_t$	-0.780	-1.496	0.135	-0.664	-1.043	0.297	-0.053	-0.068	0.946	-0.439	-0.607	0.544	0.446	0.636	0.525
$\Delta x_t$	-1.880	-2.619	0.009	-0.647	-0.884	0.377	-2.093	-2.338	0.019	-0.499	-0.699	0.485	-0.486	-0.682	0.495
	$R^2 = 0.07$			$R^2 = 0.03$			$R^2 = 0.04$			$R^2 = 0.02$			$R^2 = 0.06$		

## (ii) Heaney convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.031	1.222	0.222	0.043	1.277	0.201	0.021	0.534	0.593	0.039	1.184	0.236	0.041	1.315	0.189
$\Delta s_t$	0.214	1.196	0.232	0.463	2.907	0.004	0.370	2.211	0.027	0.258	1.429	0.153	0.491	2.441	0.015
$\Delta s_{t-1}$	0.140	0.993	0.320	-0.275	-1.452	0.146	0.029	0.158	0.875	0.137	0.811	0.417	0.177	0.942	0.346
$y_t$	-0.789	-0.919	0.358	-0.257	-0.300	0.764	-0.206	-0.138	0.890	-0.623	-4.430	0.000	-0.471	-0.442	0.658
$r_t$	-0.540	-1.099	0.272	-0.575	-1.002	0.316	-0.219	-0.293	0.769	-0.154	-0.253	0.800	-0.478	-0.774	0.439
$\Delta x_t$	-1.866	-2.647	0.008	-0.646	-0.886	0.376	-2.083	-2.316	0.021	-0.398	-0.574	0.566	-0.405	-0.552	0.581
	$R^2 = 0.07$			$R^2 = 0.03$			$R^2 = 0.04$			$R^2 = 0.03$			$R^2 = 0.04$		

## (iii) West convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.026	0.945	0.345	0.038	1.140	0.254	0.027	0.683	0.494	0.009	0.222	0.825	0.063	2.151	0.031
$\Delta s_t$	0.241	1.274	0.203	0.460	2.964	0.003	0.395	2.386	0.017	0.242	1.352	0.176	0.446	2.241	0.025
$\Delta s_{t-1}$	0.066	0.414	0.679	-0.309	-1.657	0.098	0.074	0.402	0.687	0.088	0.534	0.593	0.243	1.292	0.196
$y_t$	0.215	0.292	0.771	0.353	0.572	0.567	-0.723	-1.095	0.273	0.357	0.829	0.407	-1.967	-3.301	0.001
$r_t$	-0.721	-1.488	0.137	-0.740	-1.269	0.204	-0.052	-0.068	0.946	-0.291	-0.490	0.624	0.198	0.306	0.760
$\Delta x_t$	-1.861	-2.658	0.008	-0.655	-0.905	0.365	-2.100	-2.356	0.018	-0.547	-0.766	0.444	-0.639	-0.923	0.356
	$R^2 = 0.07$			$R^2 = 0.03$			$R^2 = 0.04$			$R^2 = 0.02$			$R^2 = 0.09$		

## (d) h = 12 months ahead

Dependent variable:  $\Delta S_{t+h}$ 

## (i) Cost-of-carry convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.058	1.330	0.184	0.084	1.591	0.112	0.056	0.899	0.368	0.052	0.938	0.348	0.111	2.209	0.027
$\Delta S_t$	-0.107	-0.424	0.672	0.228	1.010	0.313	0.595	2.348	0.019	0.184	0.615	0.538	0.605	1.908	0.056
$\Delta S_{t-1}$	-0.196	-0.775	0.438	-0.378	-1.333	0.183	0.106	0.441	0.659	-0.093	-0.365	0.715	0.099	0.353	0.724
$y_t$	1.065	0.629	0.530	1.161	0.952	0.341	-0.749	-0.716	0.474	0.969	1.644	0.100	-0.740	-0.650	0.516
$r_t$	-2.066	-2.536	0.011	-2.015	-1.993	0.046	-0.610	-0.472	0.637	-1.589	-1.538	0.124	-1.443	-1.384	0.166
$\Delta x_t$	-1.995	-2.511	0.012	-1.222	-1.216	0.224	-1.752	-1.183	0.237	-0.361	-0.333	0.739	0.063	0.052	0.958
	$R^2 = 0.07$			$R^2 = 0.04$			$R^2 = 0.03$			$R^2 = 0.03$			$R^2 = 0.05$		

## (ii) Heaney convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.070	1.788	0.074	0.095	1.811	0.070	0.057	0.908	0.364	0.084	1.655	0.098	0.106	2.154	0.031
$\Delta S_t$	-0.015	-0.062	0.950	0.330	1.404	0.160	0.574	2.315	0.021	0.236	0.776	0.438	0.569	1.795	0.073
$\Delta S_{t-1}$	-0.019	-0.080	0.936	-0.309	-1.049	0.294	0.076	0.325	0.745	-0.073	-0.274	0.784	0.085	0.298	0.766
$y_t$	-1.549	-1.098	0.272	-0.853	-0.940	0.347	-0.682	-0.384	0.701	-0.252	-0.568	0.570	-0.089	-0.101	0.920
$r_t$	-1.322	-1.732	0.083	-1.244	-1.463	0.144	-0.820	-0.675	0.499	-0.787	-0.851	0.395	-1.866	-2.063	0.039
$\Delta x_t$	-1.937	-2.557	0.011	-1.237	-1.244	0.214	-1.730	-1.176	0.240	-0.315	-0.285	0.776	0.088	0.073	0.942
	$R^2 = 0.07$			$R^2 = 0.04$			$R^2 = 0.03$			$R^2 = 0.01$			$R^2 = 0.05$		

## (iii) West convenience yield

	Al			Cu			Pb			Ni			Zn		
	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value	Coef	t-stat	p-value
Constant	0.060	1.328	0.184	0.082	1.562	0.118	0.069	1.061	0.289	0.026	0.343	0.732	0.117	2.425	0.015
$\Delta S_t$	0.046	0.178	0.859	0.320	1.439	0.150	0.614	2.422	0.015	0.219	0.728	0.467	0.554	1.762	0.078
$\Delta S_{t-1}$	-0.185	-0.683	0.494	-0.414	-1.426	0.154	0.150	0.617	0.537	-0.126	-0.496	0.620	0.121	0.422	0.673
$y_t$	0.578	0.402	0.688	1.023	1.037	0.300	-1.334	-1.532	0.126	0.732	1.082	0.279	-0.935	-1.121	0.262
$r_t$	-1.709	-2.316	0.021	-1.742	-1.920	0.055	-0.569	-0.458	0.647	-0.741	-0.800	0.424	-1.530	-1.620	0.105
$\Delta x_t$	-1.920	-2.369	0.018	-1.264	-1.269	0.204	-1.764	-1.193	0.233	-0.450	-0.415	0.678	-0.008	-0.006	0.995
	$R^2 = 0.06$			$R^2 = 0.04$			$R^2 = 0.04$			$R^2 = 0.02$			$R^2 = 0.05$		

Note: (1) In Panels (a) through (d), the estimated regression is  $\Delta S_{t+h} = \beta_0 + \beta_1 \Delta S_t + \beta_2 \Delta S_{t-1} + \beta_3 y_t + \beta_4 r_t + \beta_5 \Delta x_t + \varepsilon_t$ , where  $\Delta S_{t+h} = \ln(S_{t+h}/S_t)$  is the log-return on the spot price between t and (t + h),  $\Delta S_t$  is the log-return on the spot price between (t-1) and t,  $\Delta S_{t-1}$  is its lagged value,  $y_t$  is the convenience yield computed at time t from 3-month futures contracts,  $r_t$  is the 3-month T-Bill rate observed at time t, and  $\Delta x_t$  is the log-return on the real trade weighted U.S. Dollar index: major currencies between (t-1) and t. (2) Newey-West HAC standard errors.



**Table 3** Relative RMSFE: full sample

	Cost-of-carry				Heaney				West			
	h = 1	h = 3	h = 6	h = 12	h = 1	h = 3	h = 6	h = 12	h = 1	h = 3	h = 6	h = 12
Al	0.974	0.968	0.966	0.966	0.973	0.965	0.964	0.966	0.978	0.970	0.966	0.970
Cu	0.913	0.971	0.979	0.967	0.910	0.967	0.979	0.970	0.911	0.971	0.978	0.967
Pb	0.931	0.973	0.979	0.983	0.936	0.975	0.980	0.984	0.940	0.976	0.977	0.980
Ni	0.974	0.981	0.983	0.972	0.961	0.960	0.976	0.979	0.968	0.985	0.980	0.974
Zn	0.952	0.957	0.962	0.968	0.944	0.970	0.974	0.970	0.927	0.949	0.951	0.967

Note: Root-mean squared forecast error (RMSFE) of model (7) is relative to the random-walk model.

**Table 4** Recursive relative RMSFE

(a) Cost-of-carry

	h = 1					h = 3				
	Al	Cu	Pb	Ni	Zn	Al	Cu	Pb	Ni	Zn
Max	0.981	0.932	0.932	0.975	0.969	0.977	0.984	0.974	0.989	0.961
Min	0.965	0.901	0.889	0.950	0.939	0.950	0.957	0.951	0.970	0.912
Mean	0.974	0.916	0.919	0.966	0.956	0.964	0.971	0.967	0.980	0.946
S.E	0.003	0.009	0.011	0.007	0.007	0.005	0.008	0.005	0.005	0.011
	h = 6					h = 12				
	Al	Cu	Pb	Ni	Zn	Al	Cu	Pb	Ni	Zn
Max	0.968	0.986	0.990	0.986	0.967	0.972	0.976	0.984	0.993	0.979
Min	0.912	0.929	0.951	0.949	0.917	0.876	0.871	0.909	0.914	0.908
Mean	0.945	0.969	0.973	0.974	0.949	0.922	0.940	0.951	0.967	0.959
S.E	0.018	0.016	0.010	0.011	0.013	0.033	0.034	0.022	0.021	0.017

(b) Heaney

	h = 1					h = 3				
	Al	Cu	Pb	Ni	Zn	Al	Cu	Pb	Ni	Zn
Max	0.981	0.925	0.937	0.984	0.949	0.975	0.975	0.977	0.989	0.975
Min	0.966	0.892	0.893	0.950	0.888	0.948	0.943	0.955	0.947	0.927
Mean	0.974	0.910	0.922	0.968	0.923	0.962	0.962	0.971	0.971	0.961
S.E	0.003	0.009	0.012	0.011	0.021	0.005	0.006	0.005	0.014	0.011
	h = 6					h = 12				
	Al	Cu	Pb	Ni	Zn	Al	Cu	Pb	Ni	Zn
Max	0.966	0.986	0.990	0.995	0.980	0.971	0.976	0.985	0.985	0.981
Min	0.908	0.927	0.952	0.949	0.928	0.868	0.867	0.908	0.902	0.913
Mean	0.942	0.968	0.974	0.977	0.962	0.918	0.938	0.951	0.963	0.962
S.E	0.019	0.016	0.009	0.012	0.013	0.036	0.034	0.023	0.022	0.017

**Table 4** continued

(c) West

	h = 1					h = 3				
	Al	Cu	Pb	Ni	Zn	Al	Cu	Pb	Ni	Zn
Max	0.986	0.928	0.941	0.983	0.930	0.979	0.983	0.977	0.992	0.952
Min	0.972	0.897	0.915	0.949	0.902	0.954	0.957	0.955	0.971	0.912
Mean	0.980	0.913	0.933	0.972	0.921	0.967	0.971	0.971	0.984	0.939
S.E	0.004	0.009	0.006	0.007	0.007	0.005	0.007	0.005	0.004	0.009
	h = 6					h = 12				
	Al	Cu	Pb	Ni	Zn	Al	Cu	Pb	Ni	Zn
Max	0.968	0.987	0.985	0.996	0.957	0.975	0.977	0.980	0.996	0.978
Min	0.911	0.929	0.948	0.954	0.923	0.876	0.873	0.901	0.912	0.914
Mean	0.945	0.969	0.969	0.979	0.944	0.923	0.940	0.945	0.968	0.961
S.E	0.018	0.016	0.010	0.011	0.008	0.034	0.034	0.024	0.022	0.016

Note: (1) Root-mean squared forecast error (RMSFE) of model (7) is relative to the random-walk model. (2) The pseudo out-of-sample evaluation period covers January 1998-December 2017.